

Data Privacy and Algorithmic Inequality ^{*}

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Abstract

This paper develops a foundation for a consumer's preference for data privacy by linking it to the desire to hide behavioral vulnerabilities. Data sharing with digital platforms enhances the matching efficiency for standard consumption goods, but also exposes individuals with self-control issues to temptation goods. This creates a new form of inequality in the digital era—algorithmic inequality. Although data privacy regulations provide consumers with the option to opt out of data sharing, these regulations cannot fully protect vulnerable consumers because of data-sharing externalities. The coordination problem among consumers may also lead to multiple equilibria with drastically different levels of data sharing by consumers. Our quantitative analysis further illustrates that although data is non-rival and beneficial to social welfare, it can also exacerbate algorithmic inequality.

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The emergence of big data has yielded substantial benefits for consumers. By pooling together multi-dimensional data on their users and employing advanced big-data analytics, digital platform such as Google, Amazon, Facebook, and ChatGPT can offer them unparalleled access to desired products and services. In light of this trend, macroeconomic models (e.g., Jones and Tonetti (2020), Farboodi and Veldkamp (2020), Cong, Xie, and Zhang (2020)) have started to recognize data as a third factor contributing to economic growth alongside labor and capital. These models highlight that data is non-rival and may provide increasing returns-to-scale as a factor in production. However, they often represent the cost of acquiring consumer data as a uniform, reduced-form cost shared by consumers and firms. This approach overlooks the intricate and asymmetric privacy costs that data sharing may impose on consumers.

The collection of consumer data by digital platforms presents an unprecedented challenge to consumer privacy. As argued by the Stigler Committee Report (2019), Helberger et al. (2021), OECD (2021), and FTC (2022), consumers are especially susceptible to exploitation by digital platforms. The digitization of commerce enables these platforms to influence consumers individually, identifying and catering to each consumer's unique biases and vulnerabilities. Notably, a considerable proportion of online firms have adopted pervasive "dark patterns" that deceive or manipulate users into making unintended data-sharing and purchasing decisions.¹ Additionally, the OECD (2019) Report on Consumer Protection Policies highlights the crucial role of data privacy in protecting vulnerable consumers in the digital age.² Recently enacted data privacy regulations worldwide, such as the European Union's General Data Privacy Regulation (GDPR) in 2018 and the California Consumer Privacy Act (CCPA) by the State of California in the United States in 2020, were in part designed to protect consumer privacy through informed consent requirements.

Motivated by these observations, we develop a model to assess the impact of data sharing on both the aggregate and cross-sectional distribution of consumer welfare. Unlike existing literature that emphasizes that data privacy can act as a safeguard against price discrim-

¹A 2019 survey by the Internet Consumer Protection Enforcement Network discovered that 24% of 1,760 websites contained "dark behavioral nudges" (OECD (2021)). Mathur et al. (2019) found that 11.1% of a sample of 11,000 shopping websites employed dark patterns. They also identified 22 third-party entities that provide services for creating dark patterns. Epic Games was recently fined \$520 million by the FTC for invading children's privacy and using "dark patterns" that nudged users into making unwanted purchases in its Fortnite game.

²Legal scholars, such as Zarsky (2019) and Spencer (2020), have suggested that the core objection to online manipulation of consumers is not its manipulative nature but its implementation—intense data collection, personalization, and real-time execution made possible by the internet. For example, TikTok personalizes videos displayed to users based on data collected from previous searches, fostering addictive attention allocation to the platform. This addiction has allegedly contributed to mental health issues among its younger users (<https://www.bbc.com/news/uk-wales-62720657>).

ination by firms or social discrimination by the public, we explore a new dimension: data privacy enables consumers to conceal their behavioral vulnerabilities from companies. Our analysis reveals that the capacity of online platforms and firms to profit from consumers' behavioral weaknesses represents a cost of data sharing unequally borne by all consumers. This growing disparity between those who mainly benefit from data sharing and those who are also vulnerable to exploitation and may suffer from it leads to "algorithmic inequality," a novel form of inequality in the digital era.³

We focus on limited self-control as a specific form of vulnerability affecting a consumer's data-sharing decision. Consumers with self-control issues may struggle to resist buying tempting goods, even if these goods do not benefit or even harm them. Aware of their self-control problems, these consumers may choose to keep their data private to avoid being targeted by firms selling such goods, even if it means losing access to desired products. We use the temptation utility framework developed by Gul and Pesendorfer (2001) to analyze the data-sharing choices of these consumers. Because weak-willed consumers in this framework experience mental costs from resisting temptation goods, they prefer smaller menus without such items. Their data sharing and firms' advertising influence these menus, ultimately shaping their privacy preferences on digital platforms.

Our model features an online platform, such as Google, TikTok, or Facebook, that can collect consumer data and share it with firms. There are N normal goods firms, each selling a standard consumption good like music, and J firms offering temptation goods. Each firm can target advertisements to potential buyers at a convex cost. There are two types of consumers: strong-willed consumers who always resist temptation goods, and weak-willed consumers who may indulge in temptation goods. Both strong-willed and weak-willed consumers benefit from consuming a specific normal good they prefer, while only weak-willed consumers may succumb to a specific temptation good they struggle to resist. Since each consumer prefers at most one normal good and one temptation good, the presence of consumers not interested in a specific good interferes with the firm's advertising efforts to reach its target audience.

While a consumer might prefer the platform to share her data exclusively with normal goods firms, the same data can also be shared with temptation goods firms. This non-rivalry of data is often praised as a benefit of the data-driven economy (e.g., Jones and Tonetti (2020)). However, we highlight a dimension in which it is problematic: it prevents exclusivity of data usage. This issue is particularly pervasive, not only because it is challenging to

³Our concept of "algorithmic inequality" differs from concerns about unequal treatment by algorithms due to statistical discrimination (e.g., Cowgill and Stevenson (2020), Cowgill and Tucker (2020)). Luguri and Strahilevitz (2021), for example, present evidence that dark patterns disproportionately affect less educated consumers, resulting in distributive consequences.

draft highly contingent data authorization agreements, but also because consumers do not internalize the externalities that stem from their data-sharing decisions.

We assume normal goods firms cannot perfectly price discriminate because both strong- and weak-willed consumers have a random utility for normal goods. As a result, both consumer types benefit from buying normal goods and prefer receiving advertisements from the firm that produces their favorite normal good. Strong-willed consumers, who can resist temptation goods, does not mind receiving temptation goods ads. As a result, they prefer larger menus of goods and sharing their data for more precise targeting by their favorite normal goods firm.

Data sharing presents a more complex trade-off for weak-willed consumers, because they benefit from better targeting by normal goods firms but also face increased exposure to temptation goods firms' advertisements. This central tension in our model affects social welfare under different data-sharing schemes. Aware of their vulnerability, weak-willed consumers may decide to protect themselves by opting not to share data when given the choice.⁴

We first examine two benchmark data-sharing schemes: one without any data sharing where consumers remain anonymous to firms, and one with full data sharing where firms can perfectly observe each consumer's type. In the no-data-sharing scheme, resembling traditional advertising, each firm encounters a dark pool of consumers and can only advertise to a random subset due to advertising costs. This protects weak-willed consumers from temptation goods but restricts access to normal goods for all consumers. In the full-data-sharing scheme, all firms can accurately target consumers. While this scheme enhances access to normal goods, weak-willed consumers become more susceptible to temptation goods. Consequently, the welfare gap between strong- and weak-willed consumers, our measure of algorithmic inequality on the platform, is consistently higher with full data sharing. If the harm from temptation goods is significant enough, full data sharing may also damage overall social welfare compared to no data sharing.

We then examine an opt-in/opt-out data sharing scheme similar to the GDPR, where consumers can choose to share or withhold data from platforms and firms. In our model, strong-willed consumers always opt in, while the nontrivial trade-off between improved access to normal goods and increased exposure to temptation goods leads weak-willed consumers

⁴In our model, the self-control utility framework implies that weak-willed consumers make fully rational data-sharing choices, despite their lack of self-control in consumption choices. This approach establishes a solid foundation for our normative analysis of data-sharing schemes and privacy regulations (e.g., Attanasio and Weber (2010)), albeit at the cost of overlooking consumers who are unaware of their vulnerabilities. For instance, the use of hyperbolic discounting may lead consumers, particularly those who are naive about their weakness, not to fully internalize their lack of self-control, as seen in works by Laibson (1997), O'Donoghue and Rabin (1999) and DellaVigna and Malmendier (2004). We exclude such naive consumers from our analysis because they would not use data-sharing choices to protect their vulnerabilities.

to follow a cut-off strategy. Highly tempted consumers opt out, while less tempted ones opt in. Their data-sharing choices consequently impact the composition of opt-in and opt-out pools faced by firms.

One might expect that due to the optimality of each consumer’s data-sharing choice, the GDPR would improve social welfare relative to both the no-data-sharing and full-data-sharing schemes. However, the comparison between these schemes is complicated by the presence of data-sharing externalities—when a consumer opts in to share her data, it allows firms to infer other consumers’ preferences. As a result, full data sharing dominates the other two schemes if weak-willed consumers’ self-control problem is sufficiently modest, no data sharing dominates if the self-control problem is sufficiently severe, and the opt-in/opt-out scheme offers the highest social welfare if the self-control problem falls within an intermediate range. Regardless of the utilitarian social welfare ranking, full data sharing consistently yields the highest welfare gap between strong- and weak-willed consumers, i.e., the worst algorithmic inequality, because it provides the least protection to weak-willed consumers.

Interestingly, under the opt-in/opt-out scheme, data-sharing externalities can result in multiple equilibria, with full data-sharing being one potential outcome when the temptation problem is modest. In this scenario, if all consumers opt-in, then the most tempted weak-willed consumers lack the protection to opt-out, and are forced to opt-in as well. The existence of such multiple equilibria suggests that minor changes in the quality of normal and temptation goods can cause significant shifts in consumers’ data-sharing decisions, leading to the most extreme form of algorithmic inequality.

We calibrate our model using 2021 e-commerce revenue and advertising cost, as well as GDPR efficacy, to assess the severity of self-control issues on online platforms. Under the calibrated model parameters, three equilibria exist: one where all weak-willed consumers opt-in, one where an intermediate fraction opt-in, and one where a minimal fraction opt-in. Across these three equilibria, the full data-sharing equilibrium delivers the highest utilitarian welfare, which is 16.6% higher than the minimum data-sharing equilibrium. However, this comes at the cost of widening algorithmic inequality between strong- and weak-willed consumers by 11.3%. Our findings confirm that while data sharing provides considerable benefits to society, it does so at a significant cost to vulnerable consumers.

Lastly, we extend our model to a dynamic setting where firms can use the data they accumulate to improve their goods over time. The opt-in/opt-out scheme now features a dynamic data-sharing externality, as a current consumer’s data-sharing decision impacts the quality of normal goods and the allure of temptation goods for future generations. We simulate this dynamic model assuming consumers coordinate on the minimum data-sharing equilibrium. Our findings show that the opt-in/opt-out scheme reduces algorithmic inequality by 13.8%,

but leads to a 15.5% decrease in social welfare compared to full data-sharing. This result further emphasizes the economic trade-off that data-sharing creates between efficiency and algorithmic inequality in the digital age.

Related Literature The data-sharing externalities we emphasize are in line with the concept of social data introduced by Acemoglu et al. (2019), Bergemann, Bonatti, and Gan (2019), and Easley et al. (2019). These papers typically envision a hypothetical data market in which a digital platform purchases personal data from consumers. Because the data sold by one consumer can reveal information about others, such data externalities may lower the equilibrium price of data, resulting in excessive data sharing. Galperti, Levkun, and Perego (2022) use a mechanism design approach to analyze data-sharing across sellers on an e-commerce platform. In contrast to symmetric consumers featured in these models, our model includes consumers with and without self-control issues, as well as firms that sell normal and temptation goods. As a result, our model can emphasize how data sharing may not only improve social welfare but also give rise to negative externalities borne by vulnerable consumers, leading to significant algorithmic inequality.

In doing so, our model contributes to the literature that analyzes how consumer data sharing can impact the macroeconomy. Studies like Jones and Tonetti (2020), Cong, Xie, and Zhang (2020), Cong et al. (2020), and Cong and Mayer (2022) highlight that data is non-rival and exhibits an increasing returns-to-scale nature, but consider the cost of data sharing as exogenous. Farboodi and Veldkamp (2020) explore the non-rivalry of data in a dynamic stochastic general equilibrium (DSGE) model with data and capital accumulation, and show how data can have long-run diminishing returns to capital. The reduced-form approaches taken by these studies to model the costs of data sharing are simple and useful for examining growth-related issues, but they do not consider the cross-sectional heterogeneity important for analyzing inequality. A notable exception is Abis and Veldkamp (2021), which estimates that the adoption of AI could contribute to income inequality by reducing the labor share by 5%. Unlike previous studies in this literature, our paper emphasizes that data-sharing introduces inequality among consumers.

Our paper brings data privacy into the extensive literature on protecting vulnerable consumers. There is substantial evidence that firms exploit vulnerable consumers in various ways, such as over-pricing leisure goods like credit card-financed consumption, under-pricing investment goods like health club memberships targeting time-inconsistent consumers (e.g., DellaVigna and Malmendier (2004, 2006)), targeting impulsive borrowers with high-interest payday loans (e.g., Bertrand and Morse (2011), Melzer (2011)), using add-on pricing to target inattentive consumers (e.g., Gabaix and Laibson (2006)), and employing overdraft

fees to extract billions from inattentive consumers (e.g., Stango and Zinman (2014)).

Digital technologies have significantly increased the capacity for firms to exploit consumer vulnerabilities, such as how fintech lenders target borrowers with self-control issues using payment information (e.g., Di Maggio and Yao (2020) for evidence and He, Huang, and Zhou (2021) for a related model). Digital technologies have also led to new forms of vulnerability, such as digital addiction. Allcott et al. (2020) find evidence of the addictive nature of social media among Facebook users, while Allcott, Gentzkow, and Song (2021), by estimating a structural model through a randomized experiment, show that self-control problems contribute to 31 percent of social media use. Our analysis warns that the exploitation of consumer vulnerabilities through data-sharing has a social dimension.

Many studies have investigated how data sharing allows firms to engage in price discrimination, with comprehensive reviews provided by Acquisti, Taylor, and Wagman (2016) and Goldfarb and Tucker (2019). The impact of data sharing on social welfare with price discrimination often depends heavily on the competitive landscape. For example, Taylor (2004) and Acquisti and Varian (2005) conclude that it is optimal for sellers to use consumers' past purchase data for price discrimination when consumers are naive about how their information is used, but not when consumers are sophisticated and can adjust their purchasing strategies accordingly. Ali, Lewis, and Vasserman (2019) examine how consumer data disclosure choices can intensify competition between firms in a competitive environment and prompt price concessions from a seller in a monopolistic setting. Ichihashi (2020) demonstrates that a multi-product seller may prefer not to use consumer information for pricing, allowing consumers to reveal their preferences and enabling the seller to recommend the most suitable product matches. Since price discrimination shifts social surplus towards firms without necessarily causing a deadweight social loss, we argue that protecting vulnerable consumers presents a more persuasive rationale for addressing algorithmic inequality and advocating for privacy protection.

The literature, such as Ali and Benabou (2020) and Jann and Schottmuller (2020), has also emphasized social discrimination as another reason for individuals to value privacy. When an individual's actions are publicly observable, the public may use these actions to infer the individual's unobservable characteristics, which in turn promotes pro-social behavior and discourages the individual from acting based on her private information and personal preferences. Tirole (2021) further argues that without privacy protections, political authorities could implement a social rating system that combines each individual's political stance and social network to exert control over society without resorting to harsh repression or spreading misinformation. In contrast, vulnerable consumers in our setting value privacy to avoid exploitation of their behavioral weaknesses rather than because of discrimination.

1 A Model of Data Sharing

We examine a digital platform’s ecosystem where consumers can purchase goods from firms on the platform. The process consists of two stages. First, consumers join the platform and decide if they want to share their data. In the second stage, the digital platform collects consumers’ digital history, such as past searches and purchases, subject to their consent if applicable, and passes on the data to firms.⁵ We assume this stage lasts multiple years, allowing the platform to gather sufficient data to infer consumers’ preferences. During this stage, consumers can purchase goods from firms that target potential buyers based on the information they possess about the consumers.

There are $N > 1$ normal goods, such as music and clothing, which are desirable to all consumers. Following the approach of Ichihashi (2020), we assume there are multiple types of goods, and each consumer only desires one of those goods. As a result, firms face a nontrivial matching problem with consumers, and data sharing by consumers helps to improve the matching efficiency. There are also J temptation goods in the ecosystem.⁶ We index the N normal goods by $n \in \{1, \dots, N\}$, and the J temptation versions of normal goods by $j \in \{N + 1, \dots, N + J\}$. There are two types of consumers: strong-willed consumers without self-control issues and weak-willed consumers who are subject to them.

Following Jones and Tonetti (2020), we assume data is non-rival in that that the platform can share consumer data with both normal and temptation good firms, and cannot commit to sharing data with only normal good firms.⁷ This creates a cost for consumers when sharing data. However, unlike Jones and Tonetti (2020), this cost is endogenous and affects consumers differently. For strong-willed consumers, the cost is zero; for weak-willed consumers, the cost is positive and increases with the severity of their self-control issue. This reflects the complex nature of data sharing, where consumers cannot control how their data is used, and sharing data for beneficial purposes does not prevent it from being used for harmful ones.

⁵We implicitly assume the platform has perfect data security so that consumers do not face the risk that their data may be hacked by an adversarial third-party. See Fainmesser, Galeotti and Momot (2021) for an analysis of how a platform’s data security and data collection strategies may jointly affect a user’s activity on the platform, subject to the endogenous risk of third-party hacks.

⁶Although our model interprets the goods as merchandise, one could also view normal goods more broadly as conveniences offered by the platform to attract users, like the free search provided by Google or the free news feeds provided by Twitter. On the other hand, temptation goods can represent potential harm that data sharing might cause, such as unnecessary purchases, excessive borrowing, or addictive content.

⁷Sockin and Xiong (2023) explore how a lack of commitment by digital platforms can lead to exploitation of users. They argue the trend toward decentralization of digital platforms through tokenization helps to mitigate this conflict of interest.

1.1 Consumers

There is a total of one unit of consumers divided into two types: strong-willed and weak-willed, with π_S and π_W units, respectively, and $\pi_S + \pi_W = 1$. We make the natural assumption that there are more strong- than weak-willed consumers on the platform, i.e., $\pi_S > \frac{1}{2}$. Strong-willed consumers can resist temptation goods, while weak-willed consumers might not. Each consumer desires one normal good (n). Each weak-willed consumer is also tempted by one temptation good (j) that they might buy if advertised. We use S_n to represent the group of strong-willed consumers who want normal good n , and W_{nj} to represent the group of weak-willed consumers who prefer normal good n and is tempted by temptation good j .

The presence of multiple groups of weak-willed consumers in the population is important to our model, as it serves as potential camouflage for weak-willed consumers in the opt-in/opt-out data-sharing scheme that we will analyze later. We focus on a symmetric equilibrium, assuming preferences are uniformly distributed across goods. This implies that each consumer has an equal chance of preferring any of the N normal goods, and each weak-willed consumer has an equal chance of desiring any of the J temptation goods. Both strong- and weak-willed consumers may choose to consume any good based on their preferences and the advertisements they receive from firms.

To abstract from the interaction between data sharing and wealth distribution, we assume consumers pool their wealth, as in a Lucas household, and fully insure each other against menu risk. As a result, a consumer does not face a budget constraint, and can choose to consume any advertised good, even though they have preferences for at most one normal and one temptation good.⁸ In other words, each consumer can choose any good independently, even if they are advertised multiple goods.⁹

To establish a microfoundation of data-sharing costs, we use the framework developed by Gul and Pesendorfer (2001), which offers an axiomatic foundation for self-control issues. In line with Kreps (1979), this framework outlines a consumer's preferences for a menu. In standard utility theories, rational consumers strictly prefer a larger menu, which offers a larger choice set and thus a greater maximum utility. In contrast, this temptation utility

⁸Self-control issues are not only a problem for disadvantaged consumers. For example, Jike Zhang, a golden boy in Chinese sports and the 2012 Olympic table tennis champion, reportedly suffered from a severe gambling problem. This led him to accrue debt in the tens of millions of dollars, despite his substantial commercial income of around \$10 million in 2017. Under pressure from debt collection, Jike Zhang even resorted to releasing private photos of his former movie-star girlfriend. This action ultimately resulted in significant losses in both his reputation and commercial endorsements. This example illustrates that self-control issues can affect individuals from various backgrounds, including celebrity and wealthy individuals.

⁹Adding a budget constraint would introduce additional distortions for a weak-willed consumer because temptation goods may then potentially crowd out her normal good consumption. We ignore this effect to concentrate on the direct impact of her data-sharing choices. When she shares data with the platform, she gains better access to normal goods at the risk of being targeted with temptation goods.

framework reflects that a larger menu with temptation choices may be undesirable to a consumer with self-control problems.

The consumer's preference for a menu M is given by the following:¹⁰

$$\max_{x \in M} [u(x) + v(x) - p(x)] - \max_{x' \in M} v(x'), \quad (1)$$

where x is a choice from menu M , and $u(x)$, $v(x)$, and $p(x)$ are the commitment utility, temptation utility, and price, respectively, of this choice. The consumer's actual choice from the menu in the second step is determined by the first maximization in Equation (1):

$$x_* = \arg \max_{x \in M} [u(x) + v(x) - p(x)],$$

which is a compromise of the commitment and the temptation utilities. As a result of the compromise, the consumer may not choose the most tempting choice from the menu. If so, that is, $x_* \neq \arg \max_{x' \in M} v(x')$, the consumer exercises self-control.

As self-control is costly to the consumer, having the most tempting choice on the menu is undesirable even if it is not eventually chosen. The last term in Equation (1), while it does not directly affect the consumer's choice from the menu, affects the consumer's preference for the menu. Specifically, the difference between the temptation utility of the actual choice x_* and the maximal temptation from the menu, $\max_{x' \in M} v(x') - v(x_*)$, represents the cost of self-control incurred by the consumer when she resists the temptation good.¹¹

The menu M faced by a consumer is random and depends on firms' advertising strategies, which, in turn, depend on the platform's data-sharing scheme and the consumer's data-sharing choice. Our analysis thus builds on the random Gul-Pesendorfer temptation utility of Stovall (2010), which can also be viewed as a special case of the random Strotz (1955) utility characterized by Bénabou and Pycia (2002) and Dekel and Lipman (2012). Thus, a consumer's ex ante utility is the expected utility from all potential menus.

Temptation utility A consumer, with type $\tau_i \in \{S_n, W_{nj}\}$, has the following commitment and temptation utilities from consuming normal goods n and $n' \neq n$ and temptation goods

¹⁰In this specification, we follow Gul and Pesendorfer (2004) to exclude goods prices from temptation utilities. One may, however, argue that more expensive temptation goods are less tempting, all else being equal. Such a consideration can be incorporated into our framework by, for instance, specifying the consumer's preference instead as:

$$\max_{x \in M} [u(x) + v(x) - 2p(x)] - \max_{x' \in M} [v(x') - p(x)],$$

without qualitatively impacting our key insights. We choose the simpler specification for expositional brevity. We thank Shaowei Ke for pointing out this construction to us.

¹¹Note that this framework subsumes the standard Von Neumann-Morgenstern utility framework. That is, if $v(x) = 0$, the consumer's choice is fully determined by his commitment utility.

j and $j' \neq j$:

	strong-willed		weak-willed	
x	$u_{S_n}(x)$	$v_{S_n}(x)$	$u_{W_{n_j}}(x)$	$v_{W_{n_j}}(x)$
n	$\tilde{u}_n > 0$	0	$\tilde{u}_n > 0$	0
$n' \neq n$	0	0	0	0
j	$u_B < 0$	0	$u_B < 0$	$\gamma_i \bar{v} - u_B > 0$
$j' \neq j$	$u_B < 0$	0	$u_B < 0$	0

(2)

with $u_{\tau_i}(\cdot)$ and $v_{\tau_i}(\cdot)$ denoting the commitment and temptation utility of the consumer, respectively. Both strong- and weak-willed consumers have a random utility for normal good n , \tilde{u}_n , which has a uniform distribution $H(\tilde{u}_n) \sim U[0, \bar{u}]$, with $\bar{u} > 0$ as the maximal commitment utility of a consumer that prefers normal good n . One can interpret this random utility for the normal good as a transient taste for the good, such as desiring coffee instead of tea on a given day.

Temptation good j gives a negative commitment utility $u_B < 0$ to both strong- and weak-willed consumers, reflecting that the temptation good is ultimately harmful to consumers. As the temptation good does not give any temptation utility to strong-willed consumers (i.e., $v_{S_n}(j) = 0$), they will never buy the temptation good. Good j gives a temptation utility of $\gamma_i \bar{v} - u_B$ to a weak-willed consumer, where $\bar{v} > 0$ measures the overall temptation of weak-willed consumers to temptation good j , and $\gamma_i \in [0, 1]$ measures a consumer's degree of temptation and has a uniform distribution $G(\gamma_i) \sim U[0, 1]$ across the population of weak-willed consumers. We specify this particular form of temptation utility coefficient so that a weak-willed consumer's choice of whether to buy temptation good j , when it is on the menu, is determined by a simple expression:

$$\begin{aligned} & \max_{x \in \{j, \emptyset\}} [u_{W_{n_j}}(x) + v_{W_{n_j}}(x) - p(x)] \\ = & \max \{u_{W_{n_j}}(j) + v_{W_{n_j}}(j) - p(j), 0\} = \max \{\gamma_i \bar{v} - p(j), 0\}. \end{aligned}$$

The consumer will choose to buy good j if his temptation coefficient γ_i is sufficiently high, that is, $\gamma_i \geq p(j) / \bar{v}$. We consider γ_i to represent a general behavioral weakness that makes a weak-willed consumer susceptible to buying goods he does not need, such as with impulse buying, or, worse, goods that are harmful to purchase, such as taking on debt to finance online gaming.

It's important to note the temptation delivered by good j to a weak-willed consumer is persistent and characterized by a personalized parameter γ_i , while the commitment utility delivered by good n to a consumer (either strong-willed or weak-willed) is random. The random utility delivered by good n prevents price discrimination by firm n , even if the

firm has complete information about consumers. On the other hand, information about a weak-willed consumer allows firm j to not only precisely target its advertisements but also to price discriminate against weak-willed consumers. This asymmetric setting enables us to focus on how access to consumer data affects weak-willed consumers through their temptation utility, rather than exploring the impact of price discrimination on consumers' consumption of normal goods, which has been extensively studied in existing literature.

Menu preference The menu M that a consumer faces is determined by the advertisements the consumer receives from the firms. The menu may contain none, one, or multiple normal and/or temptation goods. It's important to note that each consumer has separate and additive utilities for consumption of normal and temptation goods. As consumer i can choose to consume any number of normal and temptation goods, we can separately denote the menu faced by the consumer for each of the goods: $\mathcal{M}_i^n \in \{\{n, \emptyset\}, \emptyset\}$ is the menu for a normal good n , with \emptyset representing the menu when good n is not advertised to the consumer and $\{n, \emptyset\}$ representing the menu when it is advertised, and $\mathcal{M}_i^j \in \{\{j, \emptyset\}, \emptyset\}$ is the menu for a temptation good j .

Then, building on the utility framework specified in Equation (1), we derive the choices of a consumer with type $\tau_i \in \{S_n, W_{n_j}\}$ from the menus $\mathcal{M}_i^{n'}$ and $\mathcal{M}_i^{j'}$:

$$\begin{aligned} x_{\tau_i}(\mathcal{M}_i^{n'}) &= \arg \max_{x \in \mathcal{M}_i^{n'}} [\tilde{u}_{\tau_i}(x) - p_{n', \tau_i}(x)], \\ y_{\tau_i}(\mathcal{M}_i^{j'}) &= \arg \max_{y \in \mathcal{M}_i^{j'}} [u_{\tau_i}(y) + v_{\tau_i}(y) - p_{j', \tau_i}(y)], \end{aligned}$$

where the prices of the two goods $p_{n', \tau_i}(x)$ and $p_{j', \tau_i}(y)$ may be discriminating, depending on the consumer's type and whether the consumer's type is known to the firms. Each consumer is competitive and takes as given the firms' advertisement policies and pricing policies.

The consumer's ex ante preference for the full menu is then

$$\begin{aligned} &U_{\tau_i} \left(\{\mathcal{M}_i^n\}_{n=1}^N, \{\mathcal{M}_i^j\}_{j=N+1}^{N+J} \right) \\ &= \sum_{n=1}^N \tilde{u}_{\tau_i}(x_{\tau_i}(\mathcal{M}_i^n)) - p_{n, \tau_i}(x_{\tau_i}(\mathcal{M}_i^n)) \\ &\quad + \sum_{j=N+1}^{N+J} u_{\tau_i}(y_{\tau_i}(\mathcal{M}_i^j)) + v_{\tau_i}(y_{\tau_i}(\mathcal{M}_i^j)) - p_{j, \tau_i}(y_{\tau_i}(\mathcal{M}_i^j)) - \max_{y \in \mathcal{M}_i^j} v_{\tau_i}(y). \end{aligned}$$

This menu preference enables us to analyze the consumer's data-sharing choice, which determines the advertisements she receives from firms and, consequently, the menu she faces.

The temptation utility that we use to represent a consumer’s self-control problem is similar to an alternative approach of using the present-bias framework developed by Laibson (1997), O’Donoghue and Rabin (1999), and DellaVigna and Malmendier (2004). Hyperbolic discounting causes a consumer to overly emphasize the enjoyment from consuming temptation goods in the present, while undervaluing the future costs, leading to self-control issues. If the consumer is sophisticated enough to anticipate her future self’s struggle with these trade-offs, she may prefer to keep temptation goods off her menu. Benabou and Pycia (2002) have shown that the temptation utility representation is equivalent to a formulation with multiple selves in conflict, making it consistent with a sophisticated consumer with present biases. We use temptation utility for its simplicity to capture the consumer’s menu preference without having to derive it from a dynamic setting, even though the present-bias framework may provide sharper positive predictions about the consumer’s choices.

By using temptation utility, our model excludes consumers who are unaware of their own self-control problems, such as naive consumers with present bias. These consumers, even when given the choice to opt-out of data sharing, would not use the choice to protect their vulnerabilities. As such, it is appropriate for our analysis to focus on the implications of data sharing for sophisticated consumers who are aware of their vulnerabilities.¹²

1.2 Firms

There are $N + J$ firms on the platform, one representative firm (firm n) for producing each of the normal goods (good n) and one (firm j) for producing each of the temptation goods (good j). For simplicity, we assume each firm faces zero marginal cost of production,¹³ but a convex cost $-c \log(1 - y)$ to advertise its goods to y units of consumers, where $c > 0$ is a constant. The convexity of the cost of advertising reflects, as in Grossman and Shapiro (1984), that it is increasingly costly to reach a broader audience.¹⁴ To the extent that consumers have limited attention and online advertisers do not want to flood them with

¹²Data-sharing choices of sophisticated consumers may affect the welfare of naive consumers with self-control problems. However, a conceptual challenge remains regarding how to account for their welfare without adopting a paternalistic approach that goes against the preferences of these consumers.

¹³This assumption allows us to abstract from firms’ production decisions and instead focus on their advertising policies.

¹⁴We can microfound this cost function. Similar to Grossman and Shapiro (1984), we can assume every advertisement the online platform sends is seen by a consumer with probability η . Then, if the firm sends out q advertisements, they are seen by a consumer with probability $1 - (1 - \eta)^q$; by the Weak LLN, exactly a fraction $y_k = 1 - (1 - \eta)^q$ of consumers see them. Define $\xi = -\log(1 - \eta)$. As a result, to reach y_k measure of consumers, firm k has to buy $q = -(1/\xi) \log(1 - y_k)$ advertisements from the platform. If the platform charges f for each advertisement, then the total cost to the firm is $f q = -(f/\xi) \log(1 - y_k) = -c \log(1 - y_k)$ for an effective cost parameter c .

unlimited advertisements, the fees have to rise progressively with the quantity.¹⁵

The normal good firm n would like to target both strong-willed and weak-willed consumers who prefer good n , but not other consumers, while a temptation good firm j aims to target only weak-willed consumers who desire good j . How much they can differentiate the consumers at time t depends on the platform’s data-sharing scheme and consumers’ data-sharing choices.

As the Lucas household receives firm profits and consumers are risk-neutral, firms will choose their policies to maximize profits. Thus, each firm $k \in \{1, \dots, N + J\}$ chooses its advertising and pricing policies based on its information set about the consumers to maximize its expected profit:

$$\Pi_k = \sup_{\{p_k, y_k\}} E \left[\int_{i \in Y_k} p_k(i) \mathbf{1}_{\{x(i)=k\}} di + c \log(1 - y_k) \mid \mathcal{I}^k \right],$$

where Y_k is the set of consumers to which firm k advertises its good, $p_k(i)$ is the price that the firm charges consumer i , $\mathbf{1}_{\{x(i)=k\}}$ is a dummy variable for indicating whether consumer i purchases good k , and y_k is the measure of the set Y_k . We assume if the firm does not advertise to a consumer, then its good is not on that consumer’s menu. Each firm is strategic and can only condition its advertisement and pricing policies on its information set \mathcal{I}^k , which may allow the firm to charge different consumers different prices.

Firms face the following participation constraints:

$$p_n \leq \bar{u}, \quad p_j \leq \bar{v}.$$

Violating these price constraints would lead to no sales.

1.3 Rational Expectations Equilibrium

We examine the effects of different data-sharing schemes on consumers and firms, without considering the platform’s incentives. We implicitly assume that the platform will share all consumer data with firms as long as it adheres to each consumer’s sharing preferences, if applicable. In Section 2, we first analyze two straightforward data-sharing schemes: one without any sharing and the other with full sharing. In both cases, consumers don’t have individual choices regarding data sharing. In Section 2.4, we investigate the GDPR-inspired scheme that allows each consumer to decide whether to share data with the platform, which then shares the authorized data with firms.

¹⁵See Chen (2022) for a model of online advertising targeting consumers with limited attention, which creates incentives for a digital platform to enhance its online content to attract consumer attention.

Under each of these data-sharing schemes, an equilibrium on the platform is a set of optimal advertising and pricing policies $\{Y_k, p_k\}$ for each firm $k \in \{1, \dots, N + J\}$, and a data-sharing choice s_i and an optimal purchase policy correspondence $\{x_\tau(\mathcal{M}_i^n), y_\tau(\mathcal{M}_i^j)\}$ for each consumer i such that the following are satisfied:

- Consumer optimization: Given each firm's advertising and pricing policies, each consumer i finds it optimal to first adopt the data-sharing choice s_i and then follow the purchase policy $\left\{ \{x_\tau(\mathcal{M}_i^n)\}_{n=1}^N, \{y_\tau(\mathcal{M}_i^j)\}_{j=N+1}^{N+J} \right\}$ for a menu set $\{\mathcal{M}_i^n, \mathcal{M}_i^j\}$.
- Firm optimization: Given each consumer's optimal policy, each firm k finds it optimal to choose an optimal advertising policy Y_k and a pricing policy p_k for its good.

To facilitate our welfare analysis, we assume firms pay the platform for its advertising services, and consequently the costs of advertising are zero-sum transfers between firms and the platform, which is ultimately owned by consumers. Because consumer preferences are quasi-linear in the cost of their purchases, we can aggregate across consumer utility and firm and platform profits to arrive at the following utilitarian social welfare:

$$\begin{aligned}
 W = & \frac{1}{N} \sum_{n=1}^N \int \tilde{u}_n \left(\pi_S \mathbf{1}_{\{n \in \mathcal{M}_{S_n}^n \cap x_{S_n}=n\}} + \pi_W \mathbf{1}_{\{n \in \mathcal{M}_{W_n}^n \cap x_{W_n}=n\}} \right) dH(\tilde{u}_n) \\
 & + \frac{\pi_W}{J} \sum_{j=N+1}^{N+J} \int \left(u_B \mathbf{1}_{\{j \in \mathcal{M}_{W_{nj}}^j \cap x_{W_{nj}}=j\}} + (u_B - \gamma_i \bar{v}) \mathbf{1}_{\{j \in \mathcal{M}_{W_{nj}}^j \cap x_{W_{nj}}=\emptyset\}} \right) dG(\gamma_i),
 \end{aligned} \tag{3}$$

The first term captures the commitment utility of both strong-willed and weak-willed consumers from consuming normal good n . The second term for weak-willed consumers represents the social deadweight loss from consumption of the temptation good, u_B , and the cost of resisting temptation, $u_B - \gamma_i \bar{v}$, for temptation good j by those who have the temptation good on their menus but choose not to consume it.

Note from Equation (1), when a weak-willed consumer buys a temptation good, the realized temptation utility offsets the maximum temptation, resulting in zero temptation utility. The price paid doesn't affect social welfare, as it's a transfer to the firm. Welfare loss comes from the negative commitment utility u_B for weak-willed consumers who buy the good and the mental cost $u_B - \gamma_i \bar{v}$ of resisting temptation for those who have the temptation good on their menu but choose not to buy it.

The social welfare given in Equation (3) highlights a trade-off in sharing consumer data with firms. It improves the matching efficiency for normal goods, boosting social welfare, but also exposes weak-willed consumers to temptation goods, lowering welfare. This differs from typical data privacy models, which focus on how consumer data availability increases

total social surplus through better matching and alters the surplus split between consumers and firms. In our model, weak-willed consumers aren't hiding data from temptation good firms to gain better pricing. They're trying to avoid exploitation and potential harm from those goods.

Our model emphasizes that data sharing has both benefits and costs, which are distributed differently among consumers. Strong-willed consumers gain better access to desired goods and services by sharing their data, while weak-willed consumers risk being exploited due to their behavioral vulnerabilities. A simplified cost function in a representative agent approach overlooks this important heterogeneity, and as a result, does not capture the crucial role of privacy regulation in addressing algorithmic inequality.

We measure this inequality as the welfare gap Δ , which represents the difference between the average utility of strong- and weak-willed consumers, taking into account the costs of purchasing normal and temptation goods:

$$\begin{aligned} \Delta = & \frac{1}{N} \sum_{n=1}^N \int (\tilde{u}_n - p_n) \left(\pi_S \mathbf{1}_{\{n \in \mathcal{M}_{S_n}^n \cap x_{S_n}=n\}} - \pi_W \mathbf{1}_{\{n \in \mathcal{M}_{W_n}^A \cap x_{W_n}=n\}} \right) dH(\tilde{u}_n) \\ & - \frac{\pi_W}{J} \sum_{j=N+1}^{N+J} \int \left((u_B - p_j) \mathbf{1}_{\{j \in \mathcal{M}_{W_{nj}}^j \cap x_{W_{nj}}=j\}} + (u_B - \gamma_i \bar{v}) \mathbf{1}_{\{j \in \mathcal{M}_{W_{nj}}^j \cap x_{W_{nj}}=\emptyset\}} \right) dG(\gamma_i). \end{aligned} \quad (4)$$

To anchor our welfare analysis of various data-sharing schemes, we examine the ideal outcome from the perspective of a planner who aims to maximize the social welfare in (3). Since advertising has no social cost, the planner would want normal good firms to sell their products to all strong- and weak-willed consumers. However, because temptation firms' advertisements bring costs to weak-willed consumers, whether they resist or give in to the temptation, the planner prefers these firms not to advertise to any consumer. Consequently, there is no algorithmic inequality in this ideal outcome, which is summarized as the first-best outcome below.¹⁶

Proposition 1 *In the first-best outcome, normal good firms sell their goods to all strong-willed and weak-willed consumers, and temptation good firms advertise to no consumers.*

One may argue the first-best outcome suggests a policy to simply ban the sale of temptation goods. However, it is important to note that temptation goods are often intricately

¹⁶In the first-best outcome, temptation firms don't advertise because their goods cause social harm. If instead $u_B > 0$, then temptation firms would sell their goods to all weak-willed consumers in the first-best. This is different from price discrimination models, where reducing search frictions to improve matching between firms and consumers always creates a social surplus. With data sharing, firms may gain more of this surplus, but better matching on the platform remains socially beneficial.

linked to normal goods in practice. For example, gambling may be a regular form of entertainment in moderation, but can become harmful when it fosters addiction (possibly when combined with debt). A good may therefore be considered normal consumption for some consumers but become harmful and addictive to others when modified in certain ways. Such links make it difficult to define temptation goods in the legal domain and, therefore, to ban these goods.¹⁷

2 Equilibrium Under Different Data-sharing Schemes

In this section, we examine the equilibrium of the ecosystem under three distinct data sharing schemes: no sharing, full sharing, and a scheme that allows consumers to opt in or out of data sharing. The first two schemes do not provide consumers with individual choices and serve as benchmarks for our analysis of the opt-in/opt-out scheme. Additionally, we evaluate consumer welfare under these schemes and offer a calibrated analysis to quantify welfare using a set of realistic parameters.

2.1 Consumer Choice

We initially analyze the choice of each consumer from a given menu of consumption goods. A strong-willed consumer that prefers normal good n may buy it if its price is below the consumer’s reservation value, and always refuses other goods. A weak-willed consumer that prefers normal good n and desires temptation good j may buy good n if its price is lower than his reservation value, just like a strong-willed consumer, and may buy temptation good j if his temptation coefficient γ_i is sufficiently high relative to the price of the good. The following proposition summarizes these choices, with the proof provided in the Appendix.

Proposition 2 *A strong-willed consumer that prefers normal good n with commitment utility \tilde{u}_n will purchase it if it is offered at a price below his reservation value $p_n \leq \tilde{u}_n$, and always reject other goods. A weak-willed consumer that prefers normal good n with commitment utility \tilde{u}_n and desires temptation good j with temptation coefficient γ_i will purchase good n if it is offered at a price below his reservation value $p_n \leq \tilde{u}_n$, and purchase good j if his temptation coefficient γ_i is sufficiently high relative to the offered price: $\gamma_i \geq \frac{p_j}{\tilde{v}}$.*

¹⁷It is possible to adjust our model so that each temptation good is a modified version of a normal good. This altered good becomes tempting for specific weak-willed consumers and leads to a negative commitment utility for them. Although this model variant complicates the notation for consumer inference, it does not change our model’s core mechanism.

This proposition shows that both strong- and weak-willed consumers might reject good n if their random utility for the good ends up being lower than the proposed price. This limitation prevents firm n from effectively price discriminating any consumer. As a result, all strong- and weak-willed consumers prefer to receive advertisements for good n , enabling them to take advantage of a high realization of their random utility for the good. This benefit motivates both strong- and weak-willed consumers to share their data with the platform.

Proposition 2 also shows that even when good j is on the menu of weak-willed consumers who desire it, only those with a sufficiently high temptation coefficient γ_i will buy it. Those with a modest temptation ($\gamma_i < p_B/\bar{v}$) resist it but still suffer a mental cost of $\gamma_i\bar{v} - u_B$ from exercising self-control. Those with sufficiently strong temptation buy good j and suffer from not only paying the price of p_j to purchase the good, but also from enduring the negative commitment utility of u_B that this purchase entails.

2.2 No Data Sharing

We begin by examining a benchmark scheme in which the platform neither collects nor shares any consumer data with firms. Consequently, firms lack information about any consumer's type and thus face a dark pool for their advertising. In this scenario, the probability of each firm's advertising reaching its intended consumers is equal to the unconditional probability. This setting can be interpreted as the practice prior to the era of big data. The following proposition characterizes the equilibrium.

Proposition 3 *With no data sharing (NS), there exists a unique equilibrium with the following properties:*

1. *A normal good firm n randomly advertises good n to y_n^{NS} measure of consumers:*

$$y_n^{NS} = \max \left\{ 1 - 4N \frac{c}{\bar{u}}, 0 \right\}, \quad (5)$$

at a uniform price: $p_n^{NS} = \frac{1}{2}\bar{u}$.

2. *A temptation good firm j randomly advertises good j to y_j^{NS} measure of consumers:*

$$y_j^{NS} = \max \left\{ 1 - 4 \frac{J}{\pi_W} \frac{c}{\bar{v}}, 0 \right\}, \quad (6)$$

at a uniform price: $p_j^{NS} = \frac{1}{2}\bar{v}$.

Under this benchmark scheme of no data sharing, the firms' undirected advertising results in inefficiency. Consequently, normal good firm n restricts its advertising to a small

pool of potential consumers. Equation (5) shows firm n 's advertising intensity y_n decreases with its cost parameter c , and increases with $\frac{1}{N}$ (the fraction of intended consumers in the population) and \bar{u} (which determines the price of good n). To the extent that firm n does not advertise good n to all strong-willed and weak-willed consumers that prefer the good, as necessitated by the first-best outcome, the no data sharing scheme results in inefficient matching of good n . This inefficiency prompts more data sharing. Equation (6) also shows that anonymity discourages temptation good firm j from targeting all weak-willed consumers that may purchase the good. This creates a source of welfare gain. Lacking knowledge about their consumers' reservation values, both firms charge all consumers the same prices for the goods, $p_n^{NS} = \frac{1}{2}\bar{u}$ and $p_j^{NS} = \frac{1}{2}\bar{v}$, which implies that the firms' advertisements are accepted by their intended consumers half of the time.

2.3 Full Data Sharing

We now explore a vastly different scheme in which the platform can collect consumers' data, allowing it to infer each consumer's type. This scheme can be interpreted as the practice employed by most digital platforms before the enactment of data privacy regulations. For simplicity, we assume the data collected by the platform enables it to determine not only whether a consumer is strong- or weak-willed but also each weak-willed consumer's temptation coefficient γ_i . While this assumption may overstate the current capabilities of big data analytics, the rapid advancement of innovative data analytics over the years brings us closer to this illustrative limiting case. By sharing consumer data with firms, the platform permits them to employ different advertising and pricing strategies for various consumer types, specifically targeting vulnerable consumers, as outlined by Nadler and McGuigan (2018) and the Stigler Committee Report (2019).¹⁸

As strong- and weak-willed consumers have the same preference for normal good n and their purchase decision regarding good n is not affected by the presence of any temptation good, there is no need for normal good firm n to differentiate strong-willed and weak-willed consumers. Proposition 4 derives y_n^{FS} the measure of strong- and weak-willed consumers, to whom firm n advertises its good, and the price p_n^{FS} the firm asks. Data sharing allows firm n to achieve a higher level of efficiency by avoiding advertising to consumers who would

¹⁸Our model simplifies many complexities of how platforms target vulnerable consumers in practice. By using big data in real time to identify each consumer's current mental state, a digital platform can not only detect a consumer's cognitive and affective vulnerabilities but also determine the most effective strategy to influence her decision-making when she is most susceptible. A platform may also employ strategies that proactively trigger consumer vulnerability, for example, by depleting a consumer's willpower through aggressive advertising. Social media platforms, in particular, customize nudges and content to maximize consumer engagement, occasionally bordering on inducing addictive behavior (e.g., Allcott et al. (2020)).

never buy good n . As a result of the improved efficiency, firm n advertises more under full data sharing than under no data sharing, that is, $y_n^{FS} \geq y_n^{NS}$, which in turn implies that both strong- and weak-willed consumers have a strictly higher probability of being covered by firm n . As the firm does not know the reservation value of the targeted consumers, it again charges the same price $p_n^{FS} = \frac{1}{2}\bar{u}$.

Access to consumer data also allows temptation good firm j to perfectly target weak-willed consumers. Furthermore, because firm j also observes the severity of each weak-willed customer's temptation, it will price discriminate each consumer by charging his full reservation value, $p_j(\gamma_i) = \gamma_i\bar{v}$, which is the net utility cost of resisting temptation. Such price discrimination in turn motivates the firm to concentrate its advertising only on the most tempted consumers, that is, those with γ_i higher than a threshold $\hat{\gamma}^{FS}$. As a result, full data sharing allows firm j to precisely target weak-willed consumers at greater intensity than under no data sharing, and to engage in perfect price discrimination against them.

We summarize the equilibrium in the following proposition.

Proposition 4 *With full data sharing (FS), there exists a unique equilibrium with the following properties:*

1. *Normal good firm n advertises its good to y_n^{FS} measure of strong- and weak-willed consumers that desire the good:*

$$y_n^{FS} = \min \left\{ \max \left\{ 1 - 4\frac{c}{\bar{u}}, 0 \right\}, \frac{1}{N} \right\}$$

at the same price $p_n^{FS} = \frac{1}{2}\bar{u}$.

2. *Temptation good firm j advertises its good to all weak-willed consumers that desire the good with $\gamma_i \geq \hat{\gamma}^{FS} = 1 - \frac{J}{\pi_W} y_j^{FS}(\bar{v})$, where $y_j^{FS}(\bar{v})$ is the total advertising by firm j :*

$$y_j^{FS}(\bar{v}) = \begin{cases} \frac{1+\pi_W/J}{2} - \sqrt{\left(\frac{1-\pi_W/J}{2}\right)^2 + \frac{\pi_W c}{J\bar{v}}} & \text{if } \bar{v} > c \\ 0 & \text{if } \bar{v} \leq c \end{cases},$$

and charges each consumer a price equal to his reservation utility $p_j(\gamma_i) = \gamma_i\bar{v}$.

Data sharing strictly benefits strong-willed consumers by improving their access to the normal good, but presents a trade-off for weak-willed consumers. On the one hand, they gain better access to normal goods, which enhances their welfare; on the other, they are also more exposed to temptation goods, which is harmful. As a result, the net effect is

ambiguous. As each weak-willed consumer suffers from the negative commitment utility u_B of a temptation good, the utilitarian welfare of weak-willed consumers is increasing in u_B . Proposition 5 shows that when the vulnerability of weak-willed consumers is sufficiently severe, that is, u_B is lower than a critical level, full data sharing reduces the welfare of weak-willed consumers so significantly that it reduces social welfare relative to no data sharing. In contrast, full data sharing enables temptation goods firms to target the most severely weak-willed consumers and charge them their reservation values, leading to a consistently higher welfare gap between strong- and weak-willed consumers under full data sharing.

Proposition 5 *There exists a critical level of u_B , below which full data sharing lowers social welfare relative to no data sharing. The welfare gap between strong- and weak-willed consumers is consistently higher under full data sharing.*

The comparison between no data sharing and full data sharing highlights a trade-off introduced by data sharing—it improves the matching efficiency of normal goods with their intended consumers at the expense of exposing weak-willed consumers to temptation goods. This trade-off leads to the implementation of data privacy regulations that allow each consumer to choose whether to opt in or out of data sharing, instead of requiring all consumers to follow the same arrangement. We explore such a scheme in the next subsection.

2.4 Opt-in/Opt-out

The European Union’s General Data Privacy Regulation (GDPR) and the California Consumer Privacy Act (CCPA) aim to safeguard consumer privacy by granting individuals the right to decide whether to share their data with digital platforms.¹⁹ These regulations have the potential to achieve a Pareto efficient outcome as each consumer can make the most suitable choice for herself.²⁰ Strong-willed consumers can opt-in and benefit from data sharing, while severely tempted consumers can opt-out to avoid exposure to temptation goods.

Before analyzing the opt-in/opt-out scheme, which we refer to as the equilibrium under GDPR, it is useful to note some important aspects of our analysis. First, strong-willed consumers benefit from sharing their data with normal good firms and are not concerned about temptation good firms. As a result, they all opt in. However, weak-willed consumers face a more complicated decision. By opting in, they gain improved access to normal goods,

¹⁹The newly enacted Virginia Consumer Data Protection Act (VCDPA) and the Colorado Privacy Act (CPA) are similar to the CCPA.

²⁰Such regulations do have an impact in practice. Goldberg et al. (2019), for instance, find that spending and visits fell by as much as 7% for EU visitors in 2018 relative to 2017 because of the GDPR, and this decline was more pronounced for smaller firms.

but also become more exposed to temptation goods. This trade-off leads more severely tempted consumers to opt-out while less tempted consumers to opt-in. Second, opting out does not guarantee full protection for vulnerable consumers. The protection provided by the opt-out pool depends on having a diverse group of consumers to act as noise in advertising for temptation goods firms. Consequently, a weak-willed consumer's decision to opt in or out may depend on other's choices, which can lead to multiple equilibria. Finally, although privacy regulations in the spirit of GDPR may seem to offer more efficient outcomes than no data sharing or full data sharing schemes, the welfare ranking remains unclear because of the externalities associated with individual data-sharing decisions.

We conjecture that weak-willed consumers with temptation coefficient γ_i higher than a critical level γ^* will choose to opt out, while those with γ_i lower than γ^* will opt in, and that the threshold is symmetric across all temptation types. Conditional on this opt-in cutoff γ^* , the utility of a weak-willed consumer that opts in with data sharing is

$$U_{W,in}^{GDPR}(\gamma_i) = \frac{y_{n,in}^{GDPR} N}{\pi_S + \pi_W \gamma^*} \int_0^{\bar{u}} \max\{\tilde{u}_{nt} - p_{n,in}^{GDPR}, 0\} dH(\tilde{u}_n) \quad (7)$$

$$+ \frac{y_{j,in}^{GDPR}(d\gamma_i) J}{\pi_W d\gamma_i} \left(u_B - p_{j,in}^{GDPR}(\gamma_i) \mathbf{1}_{\left\{\gamma_i \geq \frac{p_{j,in}^{GDPR}(\gamma_i)}{\bar{v}}\right\}} - \gamma_i \bar{v} \mathbf{1}_{\left\{\gamma_i < \frac{p_{j,in}^{GDPR}(\gamma_i)}{\bar{v}}\right\}} \right),$$

where $y_{n,in}^{GDPR}$ is the total advertising by normal good firm n to the opt-in pool at price $p_{n,in}^{GDPR}$, $y_{j,in}^{GDPR}(d\gamma_i) \in [0, \pi_W] d\gamma_i$ is the advertising intensity of temptation good firm j to opt-in consumers with temptation coefficient γ_i , and $p_{j,in}^{GDPR}(\gamma_i)$ is the price that firm j charges them. Note the consumer's utility is determined by his conditional probability of being targeted by both firms, $\frac{y_{n,in}^{GDPR}}{(\pi_S + \pi_W \gamma^*)/N}$ and $\frac{y_{j,in}^{GDPR}(d\gamma_i)}{\pi_W d\gamma_i/J}$.

His utility from opt-out, in contrast, is

$$U_{W,out}^{GDPR}(\gamma_i) = \frac{y_{n,out}^{GDPR}}{\pi_W (1 - \gamma^*)} \int_0^{\bar{u}} \max\{\tilde{u}_n - p_{n,out}^{GDPR}, 0\} dH(\tilde{u}_n) \quad (8)$$

$$+ \frac{y_{j,out}^{GDPR}}{\pi_W (1 - \gamma^*)} \left(u_B - p_{j,out}^{GDPR} \mathbf{1}_{\left\{\gamma_i \geq \frac{p_{j,out}^{GDPR}}{\bar{v}}\right\}} - \gamma_i \bar{v} \mathbf{1}_{\left\{\gamma_i < \frac{p_{j,out}^{GDPR}}{\bar{v}}\right\}} \right),$$

where $y_{n,out}^{GDPR}$ is the total advertising by normal good firm n to the opt-out pool at price $p_{n,out}^{GDPR}$, and $y_{j,out}^{GDPR}$ is the total advertising by temptation good firm j to the opt-out pool at price $p_{j,out}^{GDPR}$. For a weak-willed consumer to opt in for data sharing, it must be the case

$$U_{W,in}^{GDPR}(\gamma_i) \geq U_{W,out}^{GDPR}(\gamma_i),$$

with equality for the marginal consumer who is indifferent between opt-in and opt-out.

The following proposition characterizes the equilibrium under the GDPR. Let

$$\underline{\gamma} \equiv \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{J}} \right),$$

which is the lowest value for the equilibrium cutoff γ^* . If γ^* were below $\underline{\gamma}$, temptation good firms would not target the opt-in pool, thus invalidating such γ^* as the equilibrium cutoff.

Proposition 6 *There exists an equilibrium under the GDPR with the following properties:*

1. *All strong-willed consumers opt in, and a weak-willed consumer chooses to opt in if $\gamma_i \leq \gamma^*$ and opt out if $\gamma_i > \gamma^*$.*
2. *Normal good firm n charges the same price for the opt-in and opt-out pools: $p_{n,in}^{GDPR} = p_{n,out}^{GDPR} = \frac{1}{2}\bar{u}$, and uses a water-filling advertising strategy that prioritizes the opt-in pool:*

$$y_{n,in}^{GDPR} = \min \left\{ 1 - 4\frac{c}{\bar{u}}, \frac{1 - \pi_W(1 - \gamma^*)}{N} \right\},$$

$$y_{n,out}^{GDPR} = \min \{ \max \{ \pi_W(1 - \gamma^*) - 4N\frac{c}{\bar{u}}, 0 \}, \pi_W(1 - \gamma^*) \}.$$

3. *Temptation good firm j also adopts a water-filling advertising strategy. If $\gamma^* > \underline{\gamma}$, then it prioritizes the opt-in pool by targeting a measure $y_{j,in}^{GDPR}$, given in Equation (29), of the most-tempted consumers in the opt-in pool and charging their reservation utility: $p_{j,in}^{GDPR}(\gamma_i) = \gamma_i\bar{v}$. After it exhausts the most-tempted in the opt-in pool, it may also target a measure $y_{j,out}^{GDPR}$, given in Equation (30), of the consumers in the opt-out pool by charging a fixed price of $p_{j,out}^{GDPR} = \max \{ \frac{1}{2}, \gamma^* \} \bar{v}$. If $\gamma^* = \underline{\gamma}$, it prioritizes the opt-out pool by targeting a measure $y_{j,out}^{GDPR}$, given in Equation (27), of the consumers in the opt-out pool by charging $p_{j,out}^{GDPR} = \frac{1}{2}\bar{v}$, and it may also target an additional measure $y_{j,in}^{GDPR}$, given in Equation (28), of the most-tempted consumers in the opt-in pool and charging them: $p_{j,in}^{GDPR}(\gamma_i) = \gamma_i\bar{v}$.*
4. *The equilibrium cutoff $\gamma^* \geq \underline{\gamma}$ has the following properties:*

- (a) *For $\bar{v} < \frac{\bar{u}}{8} \min \{ (1 - \frac{4c}{\bar{u}})N, 1 \} + u_B$ or $\bar{v} > (1 - \frac{\pi_W}{J} + \frac{\pi_W}{J^2})^{-1} Jc$, there is a full data-sharing equilibrium in which all weak-willed consumers opt in ($\gamma^* = 1$).*
- (b) *For $\bar{v} < v_{***}$, where v_{***} is given in Equation (37), and $c > \frac{\bar{u}}{N}\pi_W\frac{1-\gamma}{4}$, there is an interior cutoff $\gamma^* \in (\underline{\gamma}, 1)$ that solves Equation (36). If, in addition, $\bar{v} >$*

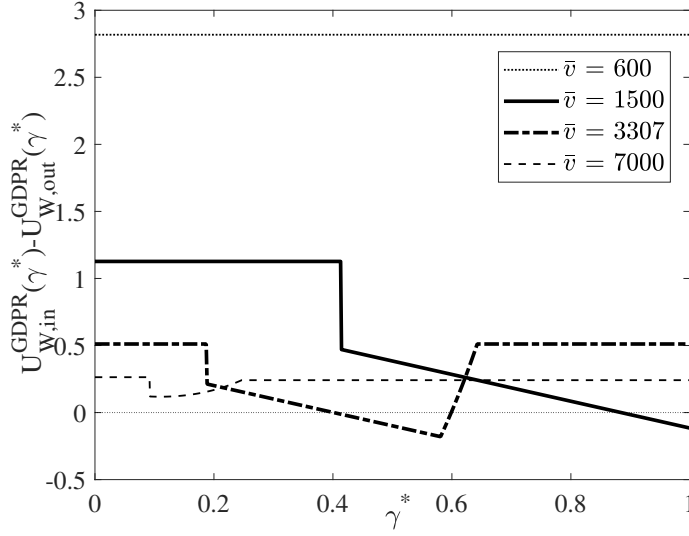


Figure 1: Illustration of the relative benefit for the marginal weak-willed consumer with temptation index γ^* to opt-in, $U_{W,in}^{GDPR}(\gamma^*) - U_{W,out}^{GDPR}(\gamma^*)$, for four values of \bar{v} . The parameters are $N = 10$, $J = 3$, $\pi_W = 0.25$, and the rest are listed in Table 1.

$(1 - \frac{\pi_W}{J} + \frac{\pi_W}{J^2})^{-1} Jc$, then there are multiple equilibria in which full data-sharing ($\gamma^* = 1$) is also an equilibrium.

Proposition 6 confirms weak-willed consumers follow a cutoff strategy when deciding whether to share their data. Temptation goods firms effectively target the more tempted consumers in the opt-in pool, charging them the full reservation value of their temptation, but do so imperfectly in the opt-out pool. The equilibrium cutoff γ^* is a complex object that depends on the incentives of temptation goods firms to search the opt-in and opt-out pools, as outlined in part 4 of Proposition 6. Figure 1 demonstrates the equilibrium by plotting the equilibrium cutoff γ^* against the net benefit for the marginal consumer to opt-in, $U_{W,in}^{GDPR}(\gamma^*) - U_{W,out}^{GDPR}(\gamma^*)$, for various values of \bar{v} . An interior equilibrium occurs when this difference is zero, a full data-sharing (all opt-in) equilibrium arises when this difference is positive for $\gamma^* = 1$, and a minimum data-sharing equilibrium ($\gamma^* = \underline{\gamma}$) occurs when this difference is negative for $\gamma^* = \underline{\gamma}$.

When \bar{v} is sufficiently small (as illustrated by the top dotted line in Figure 1 with $\bar{v} = 600$), the benefits of sharing data with normal goods firms outweigh the costs of being targeted by temptation goods firms for all weak-willed consumers. Consequently, they all opt-in, and temptation goods firms target the most tempted subset of the opt-in pool.

If \bar{v} is in an intermediate range (as illustrated by the solid black line in Figure 1 with $\bar{v} = 1500$), there is a significant benefit for weak-willed customers to opt-in. A unique equilibrium occurs where the black line intersects the x-axis. In this case, temptation goods

firms prioritize the opt-in pool for advertising and do not fully cover the opt-out pool.

When \bar{v} is sufficiently large (the thick dot and dashed line in Figure 1 with $\bar{v} = 3307$, which also corresponds to our calibration exercise in Section 2.6), there are multiple equilibria: two interior and one with full data-sharing. This occurs because the incentives for temptation good firms to search the opt-out pool increase with γ^* when γ^* is sufficiently large. This, in turn, reduces the incentives for severely weak-willed consumers to opt out, leading to a coordination problem among weak-willed consumers. As a result, full data-sharing ($\gamma^* = 1$) emerges as an equilibrium because of a coordination failure, i.e., the most tempted consumers fail to coordinate on a lower equilibrium cutoff. Alternatively, they may coordinate on two interior cutoffs. Interestingly, the minimal and intermediate data-sharing cutoffs behave very differently as \bar{v} increases (i.e., the thick dashed line shifts down). Although fewer weak-willed consumers opt-in under the minimal cutoff, more opt-in under the intermediate cutoff because of the shape of the relative benefit curve.

Finally, when \bar{v} is very large (the dashed line in Figure 1 with $\bar{v} = 7000$), all consumers opt-in once more. This occurs because temptation good firms find it profitable to fully search the opt-out pool if any mass of weak-willed consumers opt-out.

Our analysis underscores a crucial data-sharing externality. To evade targeting by temptation goods firms, the most tempted consumers may opt-out to conceal themselves in the opt-out pool. However, the opt-in decisions of other consumers, such as strong-willed and moderately tempted individuals, weaken this protection. Their departure from the opt-out pool diminishes the camouflage for the most tempted and increases the probability of weak-willed consumers in the opt-out pool being targeted by temptation goods firms. In this regard, there is a negative externality in the opt-in decisions of strong-willed and moderately weak-willed consumers because their choices do not consider the potential impact on the most vulnerable consumers.

This externality echoes the concept of social data presented by Acemoglu et al. (2019), Bergemann, Bonatti, and Gan (2019), and Easley et al. (2019), emphasizing that data has a significant social dimension because each person's data can also reveal information about others. The existence of this externality suggests that merely allowing consumers to opt in or out of data sharing may not be enough to sufficiently protect vulnerable consumers.

The existence of multiple equilibria with different cutoffs γ^* is a strong manifestation of this data-sharing externality. When \bar{v} is in an intermediate region, there can be multiple cutoffs that align with each consumer's optimal data sharing, including full data-sharing. Such multiple equilibria and the related coordination issues do not arise in either models with reduced-form cost functions for data sharing (e.g. Jones and Tonetti (2020)) or models of data markets (e.g., Acemoglu et al. (2019), Bergemann, Bonatti, and Gan (2019), and

Easley et al. (2019)).

The varying impact of the data sharing externality on strong- and weak-willed consumers further highlights the presence of algorithmic inequality on the platform. Data sharing benefits strong-willed consumers but can harm weak-willed consumers by reducing the camouflage in the opt-out pool and thereby exacerbating their temptation problem.

Empirical evidence on how consumers make their data-sharing choices is still relatively scarce because of the lack of detailed individual-level data. A notable finding in this area is the so-called data privacy paradox. Several studies, e.g., Gross and Acquisti (2005), Goldfarb and Tucker (2012), and Athey et al. (2017), have found through survey data that although consumers often express concerns about data privacy, they still tend to share their data with firms and digital platforms. This paradox is often attributed in the literature, as recently reviewed by Acquisti, Brandimarte, and Loewenstein (2020), to consumer biases such as present bias at the time of making data-sharing decisions. Our model offers a novel explanation for the data privacy paradox without having to resort to any biases in the data-sharing decision process. Despite being troubled by temptation goods, severely tempted consumers may still opt to share their data because the opt-out pool does not provide sufficient protection, rendering opting in the better choice.

Chen et al. (2021) recently analyze how a set of Alipay users authorize data sharing with third-party mini-programs on Alipay to exchange for their services. They confirm that users who express stronger concerns about their data sharing with mini-programs authorize data sharing with more, rather than less, mini-programs, and, interestingly, this pattern becomes more pronounced over time. Although our model is not specifically designed to explain this particular pattern, the coordination problem among consumers regarding their data-sharing decisions, as highlighted by our model, can help shed light on the observed increasing trend in consumers' data-sharing decisions.

There is also evidence that consumers' data sharing affects both the firms and the consumers themselves. De Matos and Adjerid (2021) found in a field experiment with a major telecommunications provider that sales, marketing communication effectiveness, and contractual lock-ins increased after new data authorizations because of the improved targeting of interested consumers. Furthermore, Aridor et al. (2020) demonstrated that although the number of observable consumers dropped by 12.5% for firms in the online travel industry because of GDPR's opt-in requirement, those who opted in could be more persistently identified and efficiently targeted.

2.5 Welfare Comparison

In this subsection, we compare the welfare consequences of the three data sharing schemes that we have analyzed: no data sharing, full data sharing, and the opt-in/opt-out scheme. The social welfare is determined by the aggregate utility of strong- and weak-willed consumers over the consumption goods, as indicated by Equation (3). We also compare the welfare gap from Equation (4) as measured by the difference in the welfare of strong- and weak-willed consumers.

Proposition 7 *The social ranking of full data sharing, no data sharing, and the GDPR, has the following properties:*

- *Full data sharing gives the highest social welfare if the temptation problem is sufficiently mild, that is, u_B is sufficiently close to zero.*
- *No data sharing gives the highest social welfare if the temptation problem is sufficiently severe, that is, u_B is sufficiently negative.*
- *There may exist an intermediate range of u_B such that the opt-in/opt-out scheme gives the highest social welfare.*

Regardless of the social welfare ranking, no data sharing delivers the lowest welfare gap.

Proposition 7 shows that social welfare involves striking a balance between better matching efficiency between normal goods firms and all consumers, and protecting weak-willed consumers from temptation goods firms. Full data sharing provides the best matching efficiency but offers the least protection, while no data sharing delivers the least matching efficiency but the best protection. The GDPR-style opt-in/opt-out scheme lies in between. As a result, full data sharing is preferred when temptation issues are minor (i.e., u_B is close to zero). In contrast, no data sharing is preferred when temptation issues are severe (i.e., u_B is significantly negative). No data sharing minimizes the welfare gap between consumer types by reducing harm from temptation goods. The opt-in/opt-out scheme might be preferred for intermediate u_B values, balancing the benefits and costs of data sharing, but could face coordination issues if too many weak-willed consumers opt in. When there are multiple equilibria under the opt-in/opt-out scheme, the equilibrium on which weak-willed consumers coordinate will influence the specific intermediate region of u_B where the opt-in/opt-out scheme delivers the highest social welfare. However, this does not impact the general ranking of the three schemes.

Parameter	Value	Target	Data	Model
π_W	0.25			
c	619.7	Advertising Revenue	821.26	821.26
\bar{u}	13524.8	Normal Goods Firm Revenue	3381.2	3381.2
\bar{v}	3306.9	Temptation Goods Firm Revenue	396.7	396.7
u_B	-364	Equilibrium Cutoff γ^*	0.40	0.40

Table 1: This table displays the parameters for our numerical experiment, the data moments to which they are targeted, and the model simulated values of these targets.

2.6 A Calibration Exercise

We now present a calibration exercise to examine consumer welfare within the opt-in/opt-out data-sharing scheme. As shown by Proposition 6, there may be one or three possible cutoff equilibria, depending on the model parameters. Our calibration exercise aims to analyze the possible equilibria under parameters calibrated to match a realistic economic environment.

2.6.1 Parameters

We calibrate our model to reflect the online spending by U.S. consumers in one year. In light of the data-sharing practices in the U.S. in 2021 (i.e., with limited regulations on firms' use of consumer data), we assume all consumers share their data. The calibrated parameters are given in Table 1.

We set the number of normal goods N to 10, and that of temptation goods J to 3. We set the fraction of weak-willed consumers π_W to 0.25 based on the ranges identified in survey data by Ameriks et al. (2007) and Toussaert (2018), who estimate that 10-30% and 23-36% of respondents have self-control issues, respectively. We set c to match the total U.S. annual online advertising revenue in 2021, amounting to \$189.3 billion as reported by IAB's 2022 Internet Advertising Revenue Report. We normalize this number by the number of U.S. online shoppers (approximately 70% of U.S. shoppers, or 230.5 million) to obtain the per-consumer value of \$821.26.²¹ We carry out this normalization for all our statistics to match the unit mass of consumers in our model.

To calculate \bar{u} and \bar{v} , we use the U.S. total annual e-commerce revenue in 2021, which amounts to \$870.8 billion, as reported by the U.S. Census Bureau (2022). We normalize this figure by dividing it by the number of U.S. online shoppers, resulting in a per capita annual revenue of \$3,777.90. Assuming 25% (i.e., π_W) of this annual expenditure is attributed to weak-willed consumers, \$3,381.20 comes from all strong-willed consumers and \$944.80 from

²¹See, for instance, <https://www.tidio.com/blog/online-shopping-statistics/>.

Equil. Cutoff	Strong-Willed		Weak-Willed		Gap	Utilitarian
	aggregate	per consumer	aggregate	per consumer		
$\gamma^* = 1.0$	1267.9	1690.6	-46.8	-187.1	1314.7	4999.1
$\gamma^* = .60$	1267.9	1690.6	19.4	77.7	1248.5	4516.7
$\gamma^* = .40$	1267.9	1690.6	87.2	348.8	1180.8	4288.7

Table 2: This table displays strong- and weak-willed consumer welfare, their difference, and utilitarian welfare for the full, opt-in/opt-out, and no data-sharing schemes for the parameters listed in Table 1.

all weak-willed consumers. We set \bar{u} to match the revenue that normal good firms earn from strong-willed consumers, which is \$3,381.20. To determine \bar{v} , we aim for the revenue that temptation goods firms receive from weak-willed consumers to be 42% of the \$944.80 they spend annually. This aligns with survey evidence from Slickdeals.net, which indicates that approximately 42% of monthly consumer spending (excluding bills such as mortgage, rent, and utilities) surpasses budgeted expenditures.

Lastly, we choose u_B so that the fraction of all consumers who would opt out if the U.S. transitioned from a full data-sharing to a minimal data-sharing opt-in/opt-out equilibrium (i.e., the equilibrium with the lowest γ^*) is 15% lower. This corresponds to the 15% reduction in web traffic resulting from the implementation of the GDPR, as measured by Congiu et al. (2022). We focus on this indirect measure of consumer opt-outs because privacy preferences are malleable, making direct evidence challenging to interpret. This approach implies a cutoff value of 0.40 for γ^* in this minimal data-sharing equilibrium.

2.6.2 Consumer Welfare

Interestingly, under our calibrated parameters from Table 1, there are three possible opt-in/opt-out equilibria, and these equilibria, as illustrated by the dashed line in Figure 1, correspond to the cutoffs of .40, .60, and 1.00 (i.e., full data sharing), respectively, for the marginal consumer. For each of these three equilibria, Table 2 compares the welfare of strong-willed and weak-willed consumers, both in the aggregate and per consumer. Since consumers have linear utility, these welfare values can be interpreted as the dollar surpluses that accrue to each group.

Across the three equilibrium, the utilitarian welfare is monotonically increasing with the extent of data sharing among the consumers—the social welfare is highest for the equilibrium with full data sharing ($\gamma^* = 1.00$) and lowest for the equilibrium with the least data sharing ($\gamma^* = 0.4$), with a difference of 16.6%. This pattern aligns with Jones and Tonetti (2020), which emphasizes that because of the non-rival nature of data, greater data sharing enhances social welfare.

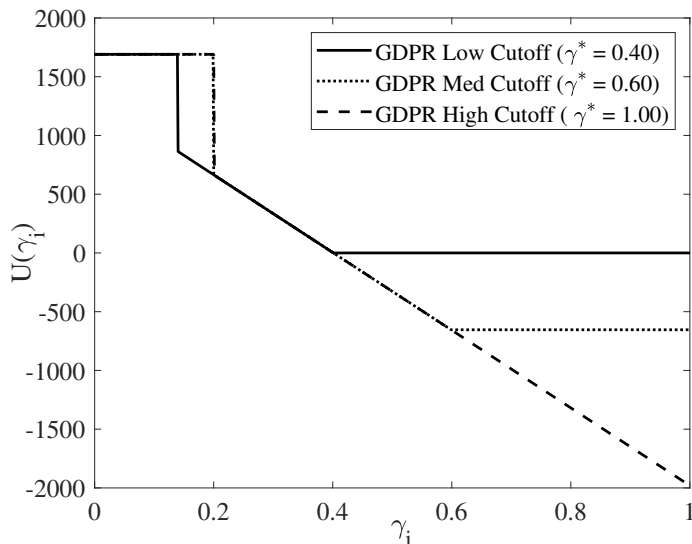


Figure 2: Heterogeneous welfare among weak-willed consumers under the three possible (High, Medium, and Low) GDPR cutoff equilibria for the parameters listed in Table 1.

Even though social welfare is increasing with data sharing, the gain is not uniformly distributed across the groups, as reflected by the following observations. First, the welfare of strong-willed consumers remains the same across the three possible equilibria. This is because under the calibrated advertising cost c , which is relatively low, normal good firms always cover all consumers in the opt-in pool, regardless of the equilibrium cutoff.

Second, the welfare of weak-willed consumers monotonically declines with the extent of data sharing—Weak-willed consumers have the lowest aggregate utility of -46.8 under the full data-sharing equilibrium with $\gamma^* = 1.00$, and the highest aggregate utility of 87.2 under the equilibrium with the lowest cutoff of $\gamma^* = 0.4$. This occurs because increased data sharing exposes more weak-willed consumers to temptation goods. Consequently, the welfare gap between strong- and weak-willed consumers is largest under the full data-sharing equilibrium ($\Delta = 1314.7$) and smallest under the least data-sharing equilibrium ($\Delta = 1180.8$), with a difference of 11.3%.

Third, the utilitarian welfare encompasses not only the well-being of strong-willed and weak-willed consumers but also the payoffs of the firms. As a result, the increasing relationship between social welfare and data sharing is driven by the increasing gain of firms, despite the decreasing welfare of weak-willed consumers with data sharing. Increased data sharing allows temptation goods firms to more effectively target weak-willed consumers, leading to higher sales of temptation goods.

Finally, there is also substantial welfare heterogeneity among weak-willed consumers. Figure 2 illustrates how the various equilibria impact the welfare of each weak-willed con-

sumer. The full data-sharing equilibrium with $\gamma^* = 1.00$ (represented by the dashed line) benefits the least tempted weak-willed consumers, but significantly harms the most tempted, as evidenced by its downward slope. The least data-sharing equilibrium with $\gamma^* = 0.40$ (indicated by the solid line) provides an improvement over the full data-sharing equilibrium by setting a floor at 0 for the right tail of weak-willed consumers, because neither the normal nor temptation goods firms advertise to the opt-out pool in this equilibrium. The intermediate data-sharing equilibrium with $\gamma^* = 0.60$ (illustrated by the dotted line) increases welfare for the mildly weak-willed consumers, who are less targeted than under the $\gamma^* = 0.40$ cutoff, at the expense of the more severely weak-willed consumers, who are now targeted by temptation goods sellers in the opt-out pool.

As discussed in Section 2.4, these diverse equilibria stem from the coordination problem among weak-willed consumers. Consequently, even without any fundamental changes in the online environment, data sharing can lead to significant welfare costs for weak-willed consumers, particularly the most vulnerable, despite an overall social welfare gain through the emergence of multiple self-fulfilling equilibria. Our model thus highlights the importance of this welfare cost borne by vulnerable consumers, which we refer to as "algorithmic inequality".

The multiplicity induced by the coordination problem also suggests a role for default options in data privacy regulation to coordinate weak-willed consumers. Thaler and Sunstein (2008) effectively highlighted the significance of default options in influencing consumer choices and welfare, and the Stigler Committee Report (2019) also emphasized the importance of default data sharing options in protecting inattentive or biased consumers who may not make optimal choices.

3 A Dynamic Model of Data Sharing

Motivated by the analysis of Jones and Tonetti (2020) and Abis and Veldkamp (2021) on the long-term effects of data sharing on economic growth, we extend our model to a dynamic setting in this section. We demonstrate that data sharing not only helps to boost long-run growth but may also exacerbate algorithmic inequality. Specifically, we first highlight a dynamic externality of data sharing on the platform, in which today's data sharing by consumers impacts the quality of goods offered by firms to future consumers. We then use a calibrated exercise to examine the long-run implications of data sharing.

Suppose now that time is discrete with $t = 0, 1, 2, \dots$. In each period, there is a new generation of consumers to join the platform. There are two sub-periods in each date that correspond to the two stages in our static model. In the first sub-period, consumers join the

platform and make their data-sharing decisions with the platform. In the second sub-period, consumers can purchase goods from firms, which target their intended customers based on the data they receive from the platform about the consumers. Similar to our static model, we assume the platform shares the consumer data authorized by consumers with firms.

A key feature of the dynamic model is that more data allow each firm to enhance its product over time. That is, a normal good firm can improve the quality of its good, and a temptation good firm can make its good more tempting. If a normal good firm n collects data on a mass d_{nt} of the strong-willed and weak-willed consumers who prefer good n at time t , then the firm increases the quality of its good \bar{u}_t according to an AR(1) process

$$\log \bar{u}_{t+1} = (1 - \theta) \log \bar{u} + \theta \log \bar{u}_t + d_{nt},$$

where $\theta \in [0, 1]$ is the rate of mean reversion. That the impact of data on product quality decays over time reflects the idea that old data becomes obsolete, as discussed in Jones and Tonetti (2020) and Abis and Veldkamp (2021).

Similarly, data enables temptation good firms to make their products more enticing by utilizing big-data analytics to identify and exploit the behavioral vulnerabilities of their customers. For example, by analyzing the attention and clicking patterns of weak-willed consumers on their platform, these firms can tailor their marketing strategies to cater to these tendencies, enhancing the effectiveness of their offerings in attracting such consumers. In other words, if the company gathers data on a mass d_{jt} of the weak-willed customers who desire good j at time t , it can improve not only the allure of its product \bar{v}_t , but also the potential harm $u_{B,t}$, according to the following AR(1) processes:

$$\begin{aligned} \log \bar{v}_{t+1} &= (1 - \theta) \log \bar{v} + \theta \log \bar{v}_t + d_{jt}, \\ \log (-u_{B,t+1}) &= (1 - \theta) \log (-u_B) + \theta \log (-u_{B,t}) + d_{jt}. \end{aligned}$$

In each period, the equilibrium follows what is characterized in Proposition 6 for the opt-in/opt-out scheme with \bar{u} , \bar{v} , and u_B being replaced by \bar{u}_t , \bar{v}_t , and $u_{B,t}$. The fraction of weak-willed consumers that opt in γ_t^* evolves over time, and can exhibit path dependence when consumers must coordinate over multiple potential opt-in/opt-out equilibria.

Dynamic Data-sharing Externality

Through the enhancement of both normal and temptation goods' quality, data sharing by one generation of consumers may impose both positive and negative externalities on future generations of consumers. While data sharing by consumers contributes to better normal

goods tomorrow, data sharing by weak-willed consumers also contributes to more-tempting temptation goods tomorrow. Consumers do not internalize this feedback loop between their data-sharing decisions and the quality of both types of goods, which can result in more weak-willed consumers opting in tomorrow, exacerbating algorithmic inequality. This creates a virtuous cycle for consumers and normal goods firms, and a vicious cycle for consumers and temptation goods firms. Although data advances the technological frontier over time, as illustrated by Jones and Tonetti (2020) and Abis and Veldkamp (2021), such improvements are not necessarily beneficial for all consumers and may worsen algorithmic inequality.

This feedback loop also highlights a dynamic aspect of the non-rivalry of data. Because the platform cannot commit to withholding its data from temptation goods firms, normal goods firms subsidize the data accumulation of temptation goods firms through the voluntary data sharing of strong-willed and moderately tempted consumers. Conversely, temptation goods firms impede the data accumulation of normal goods firms because of the opt-out decisions of severely-tempted consumers.

A Calibrated Assessment

Under the opt-in/opt-out data-sharing scheme, multiple equilibria with vastly different levels of data sharing by consumers may exist due to the coordination problem among consumers discussed in Section 2.4. We now conduct a numerical exercise using the model parameters calibrated earlier to evaluate how various equilibrium paths might impact data accumulation and, consequently, consumer welfare in the long-run.

We initialize our economy at $t = 0$ with the parameters from Table 1, and choose an AR(1) parameter θ of 0.64 based on the observation of Abis and Veldkamp (2020) that standard accounting practices amortize data warehouses over 36 months. We then simulate the economy under the opt-in/opt-out scheme until it converges to the steady state. If multiple equilibria emerge, we assume that all consumers coordinate on the same cutoff equilibrium over time.

Table 3 presents the simulation results, showing significant differences in the steady-state across the three equilibrium paths, which includes full data sharing, intermediate data sharing and minimal data sharing, as determined by the three levels of the equilibrium cutoff of weak-willed consumers. As consumer data accumulates over time, good quality increases, particularly in the full data-sharing path where all consumers share their data. For example, normal goods are 3.7% more valuable in the full data-sharing path compared to the minimal sharing path (higher \bar{u}_∞), while temptation goods are 13.0% more tempting (higher \bar{v}_∞) and 13.0% more harmful (more negative $u_{B,\infty}$). Because of the dynamic good quality, 47% of weak-willed consumers opt-in in the steady-state of the minimal sharing path, compared

		Full Sharing	Intermediate Sharing	Minimal Sharing
	\bar{u}_∞	17855.3	17268.2	17213.8
	\bar{v}_∞	4168.2	3728.6	3689.6
	$u_{B,\infty}$	-458.8	-410.4	-406.1
	γ_∞^*	1.0	0.52	0.47
Strong-willed	aggregate	1673.9	1618.9	1613.8
	per consumer	2231.9	2158.5	2151.7
Weak-willed	aggregate	-46.1	130.0	130.8
	per consumer	-184.4	520.0	523.3
Gap	aggregate	1720.0	1488.9	1483.0
Utilitarian		6599.4	5583.7	5572.9

Table 3: This table displays the steady-state values for \bar{u}_t , \bar{v}_t , and $u_{B,t}$, strong- and weak-willed consumer welfare, their difference, and utilitarian welfare for the full, intermediate, and minimal data-sharing equilibria under the parameters in Table 1.

to 40% in the static equilibrium analyzed earlier. In contrast, only 52% of weak-willed consumers opt-in in the steady-state of the intermediate data-sharing path compared to 60% in the static equilibrium because of the behavior of the intermediate cutoff discussed in Section 2.4. Strong-willed consumers fare worse under the intermediate and minimal sharing paths, as less data is shared, leading to lower normal good quality in the long-run compared to the full data-sharing path.

Interestingly, the minimal sharing path mitigates algorithmic inequality compared to the full sharing path by reducing the welfare gap between strong- and weak-willed consumers by 13.8%, even though full data sharing results in 15.5% higher overall welfare. Data accumulation raises both utilitarian welfare and the welfare gap by 32.0% and 30.8% under the full data-sharing path and 29.9% and 25.6% under the minimal data-sharing path compared to the static equilibrium reported in Table 2. Thus, more data sharing, both within and across time, not only raises efficiency and social welfare, but also increases algorithmic inequality.

4 Conclusion

This paper employs a novel approach to analyze consumer privacy preferences, focusing on the desire to protect themselves from their own behavioral vulnerabilities. Sharing consumer data with digital platforms improves the efficiency of matching consumers with normal goods but exposes weak-willed consumers to predatory advertising for temptation goods. Data privacy regulations, like GDPR and CCPA, allow consumers to opt in or out of data sharing, but they may not provide sufficient protection for severely vulnerable consumers because of data sharing externalities. The coordination problem among consumers may also result

in multiple equilibria with drastically different levels of data sharing by consumers. In a dynamic setting, we also demonstrate that more data sharing may widen the welfare gap between strong- and weak-willed consumers, or algorithmic inequality, even though it also enhances utilitarian welfare by improving product quality in the long-run. Consequently, data sharing by consumers leads to a complex trade-off between promoting economic efficiency and exacerbating algorithmic inequality.

The data-sharing externalities highlighted in our analysis are built on the platform’s bundling of data sharing with both normal and temptation goods firms, which makes it costly for vulnerable consumers to opt out of data sharing because the benefits of sharing data with normal goods firms often outweigh the costs of sharing data with temptation goods firms. Although we model the benefits of data sharing as improved matching with normal goods firms, digital platforms in practice also provide free services, such as email, messaging, social networking, and entertainment content. Such conveniences make it particularly difficult to implement regulations that force digital platforms to unbundle consumers’ data sharing.

Given the limitations of data privacy regulations in protecting vulnerable consumers, one might argue for more direct remedies to mitigate consumer harm, such as banning temptation goods, providing legal recourse, or promoting platform competition. As previously discussed, banning temptation goods is difficult to implement because temptation goods for some consumers may be normal goods for others. Measuring harm from consumer exploitation, unlike price discrimination or fraud, is challenging. Calo (2013) notes that the current consumer protection legal system, primarily based on fraud and misrepresentation, struggles to address issues of consumer vulnerability exploitation, resulting in rare policing of such manipulation (Sunstein (2015)). Furthermore, increased competition might not benefit consumers because it can push online platforms to create more addictive content (Stigler Committee (2019), Ichihashi and Kim (2021)) and engage in exploitative practices to compete for revenue. Therefore, despite its limitations, protecting data privacy *ex ante* remains the most effective way to safeguard consumers on online platforms *ex post*.

References

- Acemoglu**, Daron, Ali Makhdoumi, Azarakhsh Malekian, and Asuman Ozdaglar (2019), Too Much Data: Prices and Inefficiencies in Data Markets, National Bureau of Economic Research Working Paper 26296.
- Acquisti**, Alessandro, Laura Brandimarte, and George Loewenstein (2020), Secrets and Likes: The Drive for Privacy and the Difficulty of Achieving It in the Digital Age, *Journal of Consumer Psychology* 30, 736–758.

- Acquisti**, Alessandro, Curtis Taylor, and Liad Wagman (2016), The Economics of Privacy, *Journal of Economic Literature* 54, 442–492.
- Acquisti**, Alessandro, and Hal R. Varian. (2005), Conditioning Prices on Purchase History, *Marketing Science* 24, 367–381.
- Allcott**, Hunt, Luca Braghieri, Sarah Eichmeyer, and Matthew Gentzkow (2020), The Welfare Effects of Social Media, *American Economic Review* 110, 629–676.
- Allcott**, Hunt, Matthew Gentzkow and Lena Song (2021), Digital Addiction, NBER Working Paper No. 28936.
- Ali**, S. Nageeb, and Roland Bénabou (2020), Image versus Information: Changing Societal Norms and Optimal Privacy, *American Economic Journal, Microeconomics* 12, 116–64.
- Ali**, S. Nageeb, Greg Lewis, and Shoshana Vasserman (2019), Voluntary Disclosure and Personalized Pricing, Working Paper.
- Ameriks**, John, Andrew Caplin, John Leahy, and Tom Tyler (2007), Measuring Self-Control, *American Economic Review*, 97, 966–972.
- Aridor**, Guy, Yeon-Koo Che, and Tobias Salz. 2020. The Economic Consequences of Data Privacy Regulation: Empirical evidence from GDPR, NBER Working Paper.
- Athey**, Susan, Christian Catalini, and Catherine Tucker (2017), The Digital Privacy Paradox: Small Money, Small Costs, Small Talk, NBER Working Paper No. 23488.
- Attanasio**, Orazio P., and Guglielmo Weber (2010), Consumption and Saving: Models of Intertemporal Allocation and Their Implications for Public Policy, *Journal of Economic Literature* 48, 693–751.
- Bénabou**, Roland, and Marek Pycia (2002), *Economic Letters* 77, 419–424.
- Bergemann**, Dirk, Alessandro Bonatti, and Tan Gan (2019), The Economics of Social Data, Working Paper.
- Bertrand**, Marianne, and Adair Morse (2011), Information Disclosure, Cognitive Biases, and Payday Borrowing, *Journal of Finance* 66, 1865–1893.
- Calo**, R. (2013), Digital Market Manipulation, *Geo. Wash. L. Rev.* 82, 995.
- Chen**, Daneil (2022), The Market for Attention, Working Paper.
- Chen**, Long, Yadong Huang, Shumiao Ouyang, and Wei Xiong (2021), Data Privacy Paradox and Digital Demand, Working Paper.
- Cong** Will, Danxia Xie, and Longtian Zhang (2020), Knowledge Accumulation, Privacy, and Growth in a Data Economy, *Management Science*, forthcoming.
- Cong** Will, Wenshi Wei, Danxia Xie, and Longtian Zhang (2020), Endogenous Growth Under Multiple Uses of Data, Working Paper.
- Cong**, Will and Simon Mayer (2022), Antitrust, Regulation, and User Union in the Era of Digital Platforms and Big Data, Working Paper.

- Congiu**, Raffaele, Lorien Sabatino, and Geza Sapi (2022), The Impact of Privacy Regulation on Web Traffic: Evidence from the GDPR, *Information Economics and Policy* 61, forthcoming.
- Cowgill**, B. and M. T. Stevenson (2020), Algorithmic Social Engineering, *AEA Papers and Proceedings*, 110, 96–100.
- Cowgill**, B. and C. E. Tucker (2020), Algorithmic Fairness and Economics, Working Paper.
- Deckel**, Eddie, and Barrton L. Lipman (2012), Costly Self-Control and Random Self-Indulgence, *Econometrica* 80, 1271–1302.
- De Matos**, Miguel Godinho, Idris Adjerid (2021), Consumer Consent and Firm Targeting After GDPR: The Case of a Large Telecom Provider. *Management Science*.
- DellaVigna**, Stefano, and Ulrike Malmendier (2004), Contract Design and Self-control: Theory and Evidence, *Quarterly Journal of Economics* 119, 353–402.
- DellaVigna**, Stefano, and Ulrike Malmendier (2006), Paying Not to Go to the Gym, *American Economic Review* 96, 694–719.
- DellaVigna**, Stefano (2009), Psychology and Economics: Evidence from the Field, *Journal of Economic Literature* 47, 315–72.
- Di Maggio**, Marco, and Vincent Yao (2021), Fintech Borrowers: Lax Screening or Cream-Skimming?, *Review of Financial Studies* 34, 4565–4618.
- Easley**, David, Shiyang Huang, Liyan Yang, and Zhuo Zhong (2019), The Economics of Data, Working Paper.
- Fainmesser**, Itay, Andrea Galeotti, and Ruslan Momot (2021), Digital Privacy, *Management Science*, forthcoming.
- Farboodi**, Maryam, and Laura Veldkamp (2020), A Model of the Data Economy, Working Paper.
- FTC Bureau of Consumer Protection** (2022), Bringing Dark Patterns to Light, *FTC Staff Report*.
- Gabaix**, Xavier, and David Laibson (2006), Shrouded Attributes, consumer Myopia, and Information Suppression in Competitive Markets, *Quarterly Journal of Economics* 121, 505–540.
- Galperti**, Simone, Aleksandr Levkun, and Jacopo Perego (2022), The Value of Data Records, *Review of Economic Studies*, forthcoming.
- Goldberg**, Samuel, Garrett Johnson, and Scott Shriver (2019), Regulating Privacy Online: An Economic Evaluation of the GDPR, Working Paper.
- Goldfarb**, Avi, and Catherine Tucker (2012), Shifts in Privacy Concerns, *American Economic Review* 102, 349–53.
- Goldfarb**, Avi, and Catherine Tucker (2019), Digital Economics, *Journal of Economic Literature* 57, 3–43.

- Gross**, Ralph, and Alessandro Acquisti (2005), Information Revelation and Privacy in Online Social Networks, Proceedings of the 2005 ACM Workshop on Privacy in the Electronic Society, 71–80.
- Gul**, Faruk, and Wolfgang Pesendorfer (2001), Temptation and Self-Control, *Econometrica* 69, 365–396.
- (2004), Self-Control and the Theory of Consumption, *Econometrica* 72, 119–158.
- He**, Zhiguo, Jing Huang, and Jidong Zhou (2021), Open Banking: Credit Market Competition When Borrowers Own the Data, NBER Working Paper #28118.
- Helberger**, Natali, Orla Lynskey, Hans-W. Micklitz, Peter Rott, Marjin Sax, and Joanna Strycharz (2021), EU Consumer Protection 2.0: Structural Asymmetries in Digital Consumer Markets, BEUC Report.
- Ichihashi**, Shota (2020), Online Privacy and Information Disclosure by consumers, *American Economic Review* 110, 569–95.
- Ichihashi**, Shota, and Byung-Cheol Kim (2021), Addictive Platforms, SSRN 3853961.
- Jann**, Ole, and Christoph Schottmüller (2020), An Informational Theory of Privacy, *Economic Journal* 130, 93-124.
- Jones**, Charles, and Chris Tonetti (2020), Nonrivalry and the Economics of Data, *American Economic Review* 110, 2819–58.
- Kreps**, David M (1979), A Representation Theorem for "Preference for Flexibility", *Econometrica*, 565–577.
- Laibson**, David (1997), Golden Eggs and Hyperbolic Discounting, *Quarterly Journal of Economics* 112, 443–478.
- Luguri**, Jamie and Lior Jacob Strahilevitz (2021), Shining a Light on Dark Patterns, *Journal of Legal Analysis* 13, 43–109
- Mathur**, Arunesh, Gunes Acar, Michael J. Friedman, Elena, Luchierini, Jonathan Mayer, Marshini Chetty, and Arvind Narayanan (2019), Dark Patterns at Scale: Findings from a Crawl of 11K Shopping Websites, *Proc. ACM Hum.-Comput. Interact.* 3.
- Melzer**, Brian (2011), The Real Costs of Credit Access: Evidence From the Payday Lending Market, *Quarterly Journal of Economics* 126, 517–555.
- Nadler**, Anthony and Lee McGuigan (2018), An Impulse to Exploit: the Behavioral Turn in Data-driven Marketing, *Critical Studies in Media Communication* 35, 151-165.
- O’Donoghue**, Ted, and Matthew Rabin (1999), Doing It Now or Later, *American Economic Review* 89, 103–24.
- OECD** (2019), Challenges to consumer Policy in the Digital Age, <https://www.oecd.org/sti/consumer/challenges-to-consumer-policy-in-the-digital-age.pdf>.
- OECD** (2021), Roundtable on Dark Commercial Patterns Online: Summary of discussion, [https://one.oecd.org/document/DSTI/CP/CPS\(2020\)23/FINAL/en/pdf](https://one.oecd.org/document/DSTI/CP/CPS(2020)23/FINAL/en/pdf).
- Sockin**, Michael and Wei Xiong (2023), Decentralization Through Tokenization, *Journal of Finance* 78, 247–299.

- Spencer**, S. B. (2020), The Problem of Online Manipulation, University of Illinois Law Review, 959–1005.
- Stango**, Victor, and Jonathan Zinman (2014), Limited and Varying consumer Attention: Evidence From Shocks to the Salience of Bank Overdraft Fees, Review of Financial Studies 27, 990–1030.
- Stigler** Committee (2019), Stigler Committee Report on Digital Platforms, University of Chicago.
- Stovall**, J. (2010), Multiple Temptations, Econometrica 78, 349–376.
- Strotz**, R. (1955), Myopia and Inconsistency in Dynamic Utility Maximization, Review of Economic Studies 23, 165–180.
- Sunstein**, C. R. (2015), Fifty Shades of Manipulation, Journal of Behavioral Marketing.
- Tang**, Huan (2019), The Value of Privacy: Evidence from Online Borrowers, Working Paper.
- Taylor**, Curtis R. (2004), consumer Privacy and the Market for Customer Information, RAND Journal of Economics 35, 631–650.
- Thaler**, Richard H., and Cass R. Sunstein (2008), Nudge: Improving Decision about Health, Wealth and Happiness, London: Penguin.
- Tirole**, John (2021), Digital Dystopia, American Economic Review 111(6), 2007–48.
- Toussaert**, Severine (2018), Eliciting Temptation and Self-Control Through Menu Choices: A Lab Experiment, Econometrica 86, 859–889.
- Zarsky**, T. Z. (2019), Privacy and Manipulation in the Digital Age, Theoretical Inquiries in Law 20(1), 157–188.

Appendix

A Proofs of Propositions

A.1 Proof of Proposition 2

We first consider a strong-willed consumer, that is, $\tau(i) = S_n$, who has the following preferences over different menus:

$$\begin{aligned}
 U_S(\{A, \emptyset\}) &= \max\{\tilde{u}_n - p_n, 0\}, \\
 U_S(\{B, \emptyset\}) &= 0.
 \end{aligned}$$

Consequently, firm A will buy good A if $\tilde{u}_n \geq p_n$.

Consider now a weak-willed consumer, $\tau(i) = W_{nj}$, with the following preferences:

$$\begin{aligned} U_W(\{n, \emptyset\}) &= \max\{\tilde{u}_n - p_n, 0\}, \\ U_W(\{j, \emptyset\}) &= u_B + \max\{-p_j, -\gamma_i \bar{v}\}. \end{aligned}$$

Choosing j from the menu $\{j, \emptyset\}$ is optimal if buying j delivers higher utility: $-p_j > -\gamma_i \bar{v}$, which is equivalent to $\gamma_i > \frac{p_j}{\bar{v}}$.

A.2 Proof of Proposition 3

Given the advertising and pricing strategies of normal good firm n , Proposition 2 implies that the quantity of goods sold by firm n is

$$Q_n^{NS} = \frac{1}{N} y_n^{NS} (1 - H(p_n^{NS}/\bar{u})), \quad (9)$$

and consequently the firm's profit net of the advertisement cost is

$$\Pi_n^{NS} = p_n^{NS} \frac{1}{N} y_n^{NS} (1 - H(p_n^{NS}/\bar{u})) + c \log(1 - y_n^{NS}). \quad (10)$$

Similarly, the quantity of goods sold by temptation good firm j is

$$Q_j^{NS} = \frac{\pi_W}{J} y_j^{NS} (1 - G(p_j^{NS}/\bar{v})),$$

and the net profit of firm j is

$$\Pi_j^{NS} = p_j^{NS} \frac{\pi_W}{J} y_j^{NS} (1 - G(p_j^{NS}/\bar{v})) + c \log(1 - y_j^{NS}).$$

Technological feasibility requires that $y_n^{NS} \geq 0$ and $y_j^{NS} \geq 0$.

The first-order condition of Equation (10) with respect to y_n^{NS} is

$$p_n^{NS} Q_n^{NS} = c \frac{y_n^{NS}}{1 - y_n^{NS}}. \quad (11)$$

Then, we have that

$$\Pi_n^{NS} = p_n^{NS} Q_n^{NS} + c \log(1 - y_n^{NS}) = c \frac{y_n^{NS}}{1 - y_n^{NS}} + c \log(1 - y_n^{NS}).$$

Similarly, the first-order condition with respect to y_j^{NS} is

$$p_j^{NS} Q_j^{NS} = c \frac{y_j^{NS}}{1 - y_j^{NS}}, \quad (12)$$

which further implies that

$$\Pi_j^{NS} = c \frac{y_j^{NS}}{1 - y_j^{NS}} + c \log(1 - y_j^{NS}).$$

The first-order conditions for the goods prices set by the two firms are

$$Q_n^{NS} = \frac{p_n^{NS}}{\bar{u}} \frac{1}{N} y_n^{NS} 1_{\{0 \leq p_n^{NS} \leq \bar{u}\}}, \quad (13)$$

$$Q_j^{NS} = \frac{p_j^{NS} \pi_W y_j^{NS}}{J \bar{v}} 1_{\{0 \leq p_j^{NS} \leq \bar{v}\}}. \quad (14)$$

Note that the expected quantities sold by both firms, Q_n^{NS} and Q_j^{NS} , are nonnegative, and the net profits with respect to prices are concave, since

$$\begin{aligned} \frac{d^2 \Pi_n^{NS}}{d(p_n^{NS})^2} &= -\frac{2}{\bar{u}} \frac{1}{N} y_n^{NS} h(p_n^{NS}/\bar{u}) 1_{\{0 \leq p_n^{NS} \leq \bar{u}\}} \leq 0, \\ \frac{d^2 \Pi_j^{NS}}{d(p_j^{NS})^2} &= -2 \frac{\pi_W}{J} y_j^{NS} g(\gamma_*^{NS}) \frac{1}{\bar{v}} 1_{\{0 \leq p_j^{NS} \leq \bar{v}\}} \leq 0. \end{aligned}$$

It follows that the optimal prices will always be nonnegative. Since

$$\frac{d^2 \Pi_n^{NS}}{d(y_n^{NS})^2} = -\frac{c}{(1 - y_n^{NS})^2} < 0,$$

and $\frac{d^2 \Pi_n^{NS}}{dp_n^{NS} dy_n^{NS}} = 0$, it follows that the Hessian for firm n 's optimization with respect to (p_n^{NS}, y_n^{NS}) is negative definite and that the FOCs are sufficient.

For strong-willed consumers, there are two possibilities: $p_n^{NS} \in [0, \bar{u}]$ or $p_n^{NS} \notin [0, \bar{u}]$. If $p_n^{NS} \notin [0, \bar{u}]$, then either $p_n^{NS} = 0$ or $p_n^{NS} > \bar{u}$, neither of which generates revenue for firm n , and advertising is costly. Consequently, it must be the case that $p_n^{NS} \in [0, \bar{u}]$. Then, Equations (11) and (13) imply that $p_n^{NS} = \frac{1}{2} \bar{u}$.

Similarly, for firm j , if $p_j^{NS} \notin [0, \bar{v}]$, then either $p_j^{NS} = 0$ or $p_j^{NS} > \bar{v}$. Neither case generates any revenue, but advertising is costly. If $p_j^{NS} \in [0, \bar{v}]$, then Equations (12) and (14) imply $p_j^{NS} = \frac{1}{2} \bar{v}$.

From the FOCs for y_n^{NS} and y_j^{NS} , it then follows that

$$y_n^{NS} = 1 - \frac{4Nc}{\bar{u}}, \text{ and } y_j^{NS} = 1 - \frac{J}{\pi_W} \frac{4c}{\bar{v}}.$$

Thus, the equilibrium is unique. Note that if $y_n^{NS} \leq 0$, then firm n advertises to zero consumers. Similarly, if $y_j^{NS} \leq 0$, then firm j advertises to zero consumers.

A.3 Proof of Proposition 4

With full data sharing, firms can now separately advertise to strong-willed and weak-willed consumers. We first consider the optimal advertisement and pricing policies of normal good firm n . It shall be clear that firm n target both strong-willed and weak-willed consumers that prefer good n . We denote y_n^{FS} as the measure of strong-willed and weak-willed consumers, to which firm n advertises, and p_n^{FS} as the price the firm sets.

Proposition 2 implies that strong-willed and weak-willed consumers use the same threshold p_n^{FS}/\bar{u} in their random utility \tilde{u}_n for purchasing good n . Thus, the sales of firm n is

$$Q_n^{FS} = y_n^{FS} [1 - H(p_n^{FS}/\bar{u})],$$

and the net profit of firm n is

$$\Pi_n^{FS} = p_n^{FS} y_n^{FS} [1 - H(p_n^{FS}/\bar{u})] + c \log(1 - y_n^{FS}).$$

Following the same proof for Proposition 3, it is optimal for firm n to set a price $p_n^{FS} = \frac{1}{2}\bar{u}$. The first-order condition with respect to y_n^{FS} implies that

$$y_n^{FS} = 1 - 4\frac{c}{\bar{u}}.$$

Like before, if $1 - 4\frac{c}{\bar{u}} \leq 0$, it is optimal for the firm to advertise to no consumers. That is, $y_n^{FS} = 0$. Furthermore, if $1 - 4\frac{c}{\bar{u}} > \frac{1}{N}$, then $y_n^{FS} = \frac{1}{N}$.

We now consider the policies of temptation good firm j . Firm j will advertise only to weak-willed consumers. Since firm j can discriminate by temptation types, it will exercise first-degree price discrimination by charging a weak-willed consumer his full reservation value: $p_j^{FS}(\gamma_i) = \gamma_i \bar{v}$. It can also make its advertising strategy y_j^{FS} dependent on γ_i . Since consumers with stronger temptation are willing to pay higher prices, firm j optimally prioritizes strong

temptation consumers:

$$d\hat{z}_j^{FS}(\gamma_i) = \begin{cases} 0, & \text{if } \gamma_i < \hat{\gamma}^{FS} \\ \frac{\pi_W}{J} d\gamma_i, & \text{if } \gamma_i \in (\hat{\gamma}^{FS}, 1] \end{cases} .$$

Thus, firm j 's profit is

$$\Pi_j^{FS} = \bar{v} \int_0^1 \gamma_i y_j^{FS} (d\gamma_i) + c \log(1 - y_j^{FS}) \quad \text{with} \quad y_j^{FS} = \int_0^1 y_j^{FS} (d\gamma_i) \in [0, \pi_W/J],$$

where $\int_0^1 \gamma_i y_j^{FS} (d\gamma_i)$ is understood as a Riemann-Stieljes integral.

Note that the expected revenue of firm j reduces to $\bar{v} \int_{\hat{\gamma}_j^{FS}}^1 \frac{\pi_W}{J} \gamma_i d\gamma_i = \bar{v} \frac{\pi_W}{J} \frac{1 - (\hat{\gamma}_j^{FS})^2}{2}$, where $\hat{\gamma}_j^{FS} = 1 - \frac{y_j^{FS}}{\pi_W/J}$, since $y_j^{FS} \in [0, \pi_W/J]$. The expected revenue of firm j is then $\bar{v} y_j^{FS} \left(1 - \frac{1}{2} \frac{y_j^{FS}}{\pi_W/J}\right)$, which is determined by the firm's total advertising y_j . Consequently, we can rewrite firm j 's maximization problem as choosing y_j^{FS} :

$$\Pi_j^{FS} = \bar{v} y_j^{FS} \left(1 - \frac{1}{2} \frac{y_j^{FS}}{\pi_W/J}\right) + c \log(1 - y_j^{FS}) \quad \text{with} \quad y_j^{FS} \in [0, \pi_W/J].$$

The first-order condition for y_j^{FS} is

$$\left(1 - \frac{y_j^{FS}}{\pi_W/J}\right) \bar{v} - \frac{c}{1 - y_j^{FS}} \leq 0,$$

which has an interior solution when $\bar{v} > c$. This leads to a quadratic equation:

$$(y_j^{FS})^2 - (1 + \pi_W/J) y_j^{FS} + \frac{\pi_W}{J} \left(1 - \frac{c}{\bar{v}}\right) = 0,$$

which has the following solutions:

$$y_j^{FS} = \frac{1 + \pi_W/J}{2} \pm \sqrt{\left(\frac{1 - \pi_W/J}{2}\right)^2 + \frac{\pi_W c}{J\bar{v}}}.$$

We select the negative root because to a first-order approximation the positive root is greater than 1:

$$y_j^{FS} = \frac{1 + \pi_W/J}{2} + \sqrt{\left(\frac{1 - \pi_W/J}{2}\right)^2 + \frac{\pi_W c}{J\bar{v}}} \approx 1 + \frac{\pi_W/J}{1 - \pi_W/J} \frac{c}{\bar{v}} > 1.$$

Consequently, we have that

$$y_j^{FS} = \frac{1 + \pi_W/J}{2} - \sqrt{\left(\frac{1 - \pi_W/J}{2}\right)^2 + \frac{\pi_W c}{J\bar{v}}}.$$

Again, if this solution to the first-order condition moves outside the feasible range $[0, \pi_W/J]$, it is optimal for the firm to advertise at the corner value. Consequently, the equilibrium is again unique.

Letting $p_n^{FS} = p_n^{FS}(\bar{u})$, $p_j^{FS} = p_j^{FS}(\bar{v})$, $y_n^{FS} = y_n^{FS}(\bar{u})$ and $y_j^{FS} = y_j^{FS}(\bar{v})$, we arrive at the statement of the proposition.

A.4 Proof of Proposition 5

It is easy to verify that $y_n^{FS} \geq y_n^{NS}$. Without data sharing, the probability of a strong-willed or weak-willed consumer being covered by the producer of his desired normal good is y_n^{NS}/N ; with full data sharing, the probability is y_n^{FS} . Thus, the conditional probability of a strong-willed or weak-willed consumer being covered by the producer of his desired normal good is higher with full data sharing.

We first consider utilitarian social welfare W . Across these two schemes with and without data sharing, firm n charges the same price $p_n^{NS} = p_n^{FS} = \bar{u}/2$ for its good. From Equation (5), the social welfare under no data sharing is given by:

$$\begin{aligned} W^{NS} &= \sum_{n=1}^N \frac{1}{N} y_n^{NS} \int_{p_n}^{\bar{u}} u_n \frac{du_n}{\bar{u}} + \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} y_j^{NS} u_B \int_{p_j^{NS}/\bar{v}}^1 d\gamma_i + \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} y_j^{NS} \int_0^{p_j^{NS}/\bar{v}} (u_B - \gamma_i \bar{v}) d\gamma_i \\ &= \frac{3}{8} \bar{u} y_n^{NS} + \pi_W y_j^{NS} \left(u_B - \frac{1}{8} \bar{v} \right). \end{aligned}$$

With full data sharing, temptation good firm j can perfectly price discriminate against each targeted weak-willed consumers, and the social welfare is:

$$W^{FS} = \frac{3}{8} \sum_{n=1}^N \bar{u} y_n^{FS} + \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} u_B \int_{\hat{\gamma}_j^{FS}}^1 d\gamma_i = \frac{3}{8} N \bar{u} y_n^{FS} + J y_j^{FS} u_B.$$

It then follows:

$$W^{FS} - W^{NS} = < \pi_W \left(\frac{y_j^{FS}}{\pi_W/J} - y_j^{NS} \right) u_B + \frac{1}{8} \pi_W y_j^{NS} \bar{v} + \frac{3}{8} \bar{u} (N y_n^{FS} - y_n^{NS}) < 0,$$

if $u_B < u_{B^{**}}$, where:

$$u_{B^{**}} = -\frac{\frac{3}{\pi_W} \bar{u} (N y_n^{FS} - y_n^{NS}) + \bar{v} y_j^{NS}}{8 \left(\frac{y_j^{FS}}{\pi_W/J} - y_j^{NS} \right)}.$$

That is, social welfare is lower with full data sharing than with no data sharing.

We now consider the welfare gap Δ . From Equation (4), the welfare gap under no data sharing is given by:

$$\begin{aligned} \Delta_{NS} &= \sum_{n=1}^N \frac{\pi_S - \pi_W}{N} y_n^{NS} \int_{p_n}^{\bar{u}} u_n \frac{du_n}{\bar{u}} - \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} y_j^{NS} (u_B - p_j) \int_{p_j^{NS}/\bar{v}}^1 d\gamma_i \\ &\quad - \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} y_j^{NS} \int_0^{p_j^{NS}/\bar{v}} (u_B - \gamma_i \bar{v}) d\gamma_i \\ &= \frac{1}{8} (\pi_S - \pi_W) \bar{u} y_n^{NS} - \pi_W y_j^{NS} \left(u_B - \frac{3}{8} \bar{v} \right). \end{aligned}$$

while under full data sharing:

$$\begin{aligned} \Delta_{FS} &= \frac{1}{8} (\pi_S - \pi_W) \sum_{n=1}^N \bar{u} y_n^{FS} - \sum_{j=N+1}^{N+J} \frac{\pi_W}{J} \int_{\hat{\gamma}_j^{FS}}^1 (u_B - \gamma_i \bar{v}) d\gamma_i \\ &= \frac{1}{8} N (\pi_S - \pi_W) \bar{u} y_n^{FS} - J y_j^{FS} u_B + J \bar{v} y_j^{FS} \left(1 - \frac{1}{2} \frac{y_j^{FS}}{\pi_W/J} \right). \end{aligned}$$

It then follows:

$$\begin{aligned} \Delta_{FS} - \Delta_{NS} &= \frac{1}{8} (\pi_S - \pi_W) \bar{u} (N y_n^{FS} - y_n^{NS}) - \pi_W \left(\frac{y_j^{FS}}{\pi_W/J} - y_j^{NS} \right) u_B \\ &\quad + \pi_W \left(\frac{y_j^{FS}}{\pi_W/J} \left(1 - \frac{1}{2} \frac{y_j^{FS}}{\pi_W/J} \right) - \frac{3}{8} y_j^{NS} \right) \bar{v} \\ &\geq 0, \end{aligned}$$

and the welfare gap is always positive because the first two terms are nonnegative (recall $u_B < 0$) and the last term is strictly positive (recall temptation good firm revenue is always higher with full data-sharing).

A.5 Proof of Proposition 6

In what follows, we search for a symmetric opt-in/opt-out strategy in which all weak-willed follow the same cutoff opt-in/opt-out strategy. Specifically, we conjecture that all weak-

willed consumers will opt-in if their temptation index γ_i is less than some critical γ^* , and opt-out otherwise.

To avoid an uninteresting problem, we assume normal good firms do not have the capacity to advertise to all consumers even with data-sharing. Otherwise, there is no trade-off to opting-out for weak-willed consumers.

Firms: We first characterize the optimal strategies of both normal good and temptation good firms taking the opt-in cutoff of weak-willed consumers γ^* as given. We start with the optimal strategy of normal good firm n . Suppose that firm n advertises to $y_{n,in}^{GDPR}$ measure of strong-willed and weak-willed consumers in the opt-in pool at price $p_{n,in}^{GDPR}$ and $y_{n,out}^{GDPR}$ measure of consumers in the opt-out pool at price $p_{n,out}^{GDPR}$. Then, the firm's expected profit, by the law of large numbers, is given by

$$\Pi_n = \frac{1}{N} p_{n,out}^{GDPR} y_{n,out}^{GDPR} \left(1 - \frac{p_{n,out}^{GDPR}}{\bar{u}} \right) + p_{n,in}^{GDPR} y_{n,in}^{GDPR} \left(1 - \frac{p_{n,in}^{GDPR}}{\bar{u}} \right) + c \log (1 - y_{n,out}^{GDPR} - y_{n,in}^{GDPR}),$$

where $y_{n,out}^{GDPR} \in [0, (1 - \gamma^*) \pi_W]$ and $y_{n,in}^{GDPR} \in [0, \pi_S + \gamma^* \pi_W] / N$. Note that an advertisement to the opt-in pool reaches a strong or weak-willed consumer who desires the good with perfect precision, while one to the opt-out pool reaches a weak-willed consumer (who desires the good) at a probability of $1/N$.

If $y_{n,in}^{GDPR} > 0$ and $y_{n,out}^{GDPR} > 0$, the FOCs for $p_{n,in}^{GDPR}$ and $p_{n,out}^{GDPR}$ reveal that

$$p_{n,in}^{GDPR} = p_{n,out}^{GDPR} = \frac{1}{2} \bar{u}.$$

Then, the firm's profit becomes

$$\Pi_n = \frac{\bar{u}}{4N} y_{n,out}^{GDPR} + \frac{\bar{u}}{4} y_{n,in}^{GDPR} + c \log (1 - y_{n,out}^{GDPR} - y_{n,in}^{GDPR}).$$

The marginal profit from $y_{n,in}^{GDPR}$ is strictly higher than that from $y_{n,out}^{GDPR}$, as the advertising efficiency to the opt-in pool is higher. Thus, firm n gives higher priority to the opt-in pool.

The first-order condition with respect to $y_{n,in}^{GDPR}$ gives

$$\frac{\bar{u}}{4} - c \frac{1}{1 - y_{n,out}^{GDPR} - y_{n,in}^{GDPR}} \left\{ \begin{array}{l} < 0 \quad \text{if } y_{n,in}^{GDPR} = 0 \\ = 0 \quad \text{if } y_{n,in}^{GDPR} \in (0, \pi_S + \gamma^* \pi_W) / N \\ > 0 \quad \text{if } y_{n,in}^{GDPR} = (\pi_S + \gamma^* \pi_W) / N \end{array} \right.$$

The parameter restriction $c < \frac{\bar{u}}{4}$ ensures that $y_{n,in}^{GDPR} > 0$. As $y_{n,in}^{GDPR}$ has higher priority than $y_{n,out}^{GDPR}$, we have

$$y_{n,in}^{GDPR} = \min \left\{ 1 - 4\frac{c}{\bar{u}}, (\pi_S + \gamma^* \pi_W) / N \right\}. \quad (15)$$

If $y_{n,in}^{GDPR} = (\pi_S + \gamma^* \pi_W) / N$, the firm may have capacity to cover the opt-out pool. The first-order condition for $y_{n,out}^{GDPR}$ in this scenario gives

$$y_{n,out}^{GDPR} = \min \left\{ \max \left\{ \pi_W (1 - \gamma^*) - 4N \frac{c}{\bar{u}}, 0 \right\}, \pi_W (1 - \gamma^*) \right\}. \quad (16)$$

Since firm n gives a higher priority in advertising to the opt-in pool, we can directly prove that each strong-willed consumer would prefer opt-in to opt-out. For simplicity, we skip the proof here.

We now analyze the optimal advertising strategy of temptation firm j . Suppose that firm j advertises with intensity $y_{j,in}^{GDPR}(\gamma_i)$ to weak-willed consumers in the opt-in pool at price $p_{j,in}^{GDPR}(\gamma_i) = \gamma_i \bar{v}$ and $y_{j,out}^{GDPR}$ measure of consumers in the opt-out pool at price $p_{j,out}^{GDPR}$. Note that an advertisement to the opt-out pool reaches, with probability of $\frac{1}{J}$, a weak-willed consumer, who desires the good, and whether this weak-willed consumer buys the good or not depends on whether his temptation coefficient γ_i is above $p_{j,out}^{GDPR} / \bar{v}$. A further complication is that only weak-willed consumers with γ_i above γ^* are in the opt-out pool. Thus, the firm's profit is

$$\begin{aligned} \Pi_j &= c \log \left(1 - y_{j,out}^{GDPR} - y_{j,in}^{GDPR} \right) + \bar{v}_j \int_0^{\gamma^*} \gamma_i dy_{j,in}^{GDPR}(\gamma_i) \\ &\quad + \frac{1}{(1 - \gamma^*) J y_{j,out}^{GDPR} p_{j,out}^{GDPR}} \\ &\quad \cdot \left[\left(1 - p_{j,out}^{GDPR} / \bar{v} \right) 1_{\{p_{j,out}^{GDPR} \geq \gamma^* \bar{v}\}} + (1 - \gamma^*) 1_{\{p_{j,out}^{GDPR} < \gamma^* \bar{v}\}} \right], \end{aligned}$$

where $y_{j,out}^{GDPR} \in [0, (1 - \gamma^*) \pi_W]$ and $y_{j,in}^{GDPR} = \int_0^{\gamma^*} y_{j,in}^{GDPR}(d\gamma_i) \in [0, \gamma^* \pi_W / J]$ is the total advertisement to the opt-in pool.

If $y_{j,out}^{GDPR} > 0$, then the first-order condition for $p_{j,out}^{GDPR}$ gives the following:

$$\begin{aligned} \text{If } \gamma^* &\leq \frac{1}{2}, \quad \left(1 - 2p_{j,out}^{GDPR} / \bar{v} \right) 1_{\{p_{j,out}^{GDPR} \geq \gamma^* \bar{v}\}} = 0, \\ \text{If } \gamma^* &> \frac{1}{2}, \quad p_{j,out}^{GDPR} = \gamma^* \bar{v}. \end{aligned}$$

Thus, the optimal price satisfies

$$p_{j,out}^{GDPR} = \begin{cases} \frac{1}{2}\bar{v} & \text{if } \gamma^* \leq \frac{1}{2} \\ \gamma^*\bar{v} & \text{if } \gamma^* > \frac{1}{2} \end{cases} = \max \left\{ \frac{1}{2}, \gamma^* \right\} \bar{v}.$$

Since consumers with stronger temptation are willing to pay higher prices with $p_{j,in}^{GDPR}(\gamma_i) = \gamma_i\bar{v}$, it is optimal for firm j to prioritize consumers with higher γ_i :

$$dy_{j,in}^{GDPR}(\gamma_i) = \begin{cases} 0 & \text{if } \gamma_i < \hat{\gamma}^{GDPR} \\ \frac{\pi_W}{J} d\gamma_i & \text{if } \gamma_i \in (\hat{\gamma}^{GDPR}, \gamma^*] \end{cases}. \quad (17)$$

Therefore, the expected revenue of firm j from the opt-in pool reduces to $\bar{v} \int_{\hat{\gamma}^{GDPR}}^{\gamma^*} \frac{\pi_W}{J} \gamma_i d\gamma_i = \bar{v} \frac{\pi_W}{J} \frac{(\gamma^*)^2 - (\hat{\gamma}^{GDPR})^2}{2}$. As $\hat{\gamma}^{GDPR} = \gamma^* - \frac{y_{j,in}^{GDPR}}{\pi_W/J}$ by definition, the expected revenue of firm j from advertising to the opt-in pool is determined by the firm's total advertising to the opt-in pool $y_{j,in}^{GDPR}$: $\bar{v} y_{j,in}^{GDPR} \left(\gamma^* - \frac{1}{2} \frac{y_{j,in}^{GDPR}}{\pi_W/J} \right)$. Thus, the expected profit of firm j becomes

$$\begin{aligned} \Pi_j &= \frac{1}{(1-\gamma^*)J} \left[\frac{1}{4} - \left(\gamma^* - \frac{1}{2} \right)^2 1_{\{\gamma^* > \frac{1}{2}\}} \right] \bar{v} y_{j,out}^{GDPR} \\ &\quad + y_{j,in}^{GDPR} \left(\gamma^* - \frac{1}{2} \frac{y_{j,in}^{GDPR}}{\pi_W/J} \right) \bar{v} + c \log(1 - y_{j,out}^{GDPR} - y_{j,in}^{GDPR}), \end{aligned} \quad (18)$$

and the firm's choice reduces to choosing $y_{j,in}^{GDPR}$ and $y_{j,out}^{GDPR}$.

Which pool has priority depends on which has higher marginal revenue. The marginal revenue from the opt-in pool $\bar{v} \left(\gamma^* - \frac{y_{j,in}^{GDPR}}{\pi_W/J} \right)$ is decreasing with $y_{j,in}^{GDPR}$ and has the highest value of $\bar{v}\gamma^*$ when $y_{j,in}^{GDPR} = 0$. The marginal revenue from the opt-out pool is constant: $\frac{1}{(1-\gamma^*)J} \left[\frac{1}{4} - \left(\gamma^* - \frac{1}{2} \right)^2 1_{\{\gamma^* > \frac{1}{2}\}} \right] \bar{v}$.

The first-order condition for $y_{j,in}^{GDPR}$ is

$$\bar{v} \left(\gamma^* - \frac{y_{j,in}^{GDPR}}{\pi_W/J} \right) - \frac{c}{1 - y_{j,out}^{GDPR} - y_{j,in}^{GDPR}} \begin{cases} < 0 & \text{if } y_{j,in}^{GDPR} = 0 \\ = 0 & \text{if } y_{j,in}^{GDPR} \in (0, \pi_W\gamma^*/J) \\ > 0 & \text{if } y_{j,in}^{GDPR} = \pi_W\gamma^*/J \end{cases}, \quad (19)$$

and the first-order condition for $y_{j,out}^{GDPR}$ is

$$\frac{1}{(1-\gamma^*)J} \left[\frac{1}{4} - \left(\gamma^* - \frac{1}{2} \right)^2 1_{\{\gamma^* > \frac{1}{2}\}} \right] \bar{v} - \frac{c}{1 - y_{j,out}^{GDPR} - y_{j,in}^{GDPR}} \begin{cases} < 0 & \text{if } y_{j,out}^{GDPR} = 0 \\ = 0 & \text{if } y_{j,out}^{GDPR} \in (0, (1-\gamma^*)\pi_W) \\ > 0 & \text{if } y_{j,out}^{GDPR} = (1-\gamma^*)\pi_W \end{cases} \quad (20)$$

When $\gamma^* < \frac{1}{2}$, the opt-in pool has priority whenever $\gamma^* > \frac{1}{4} \frac{1}{(1-\gamma^*)J}$, which is equivalent to $\gamma^* \in \frac{1}{2} \left[1 - \sqrt{1 - \frac{1}{J}}, 1 + \sqrt{1 - \frac{1}{J}} \right]$, which exists and has its upper end above $\frac{1}{2}$. When $\gamma^* > \frac{1}{2}$, it is direct to verify that the opt-in pool has priority. Taken together, the opt-in pool has priority if and only if²²

$$\gamma^* > \underline{\gamma} = \frac{1}{2} \left(1 - \sqrt{1 - \frac{1}{J}} \right). \quad (21)$$

If $\gamma^* \leq \underline{\gamma}$, the opt-out pool has priority. In this case, the firm first targets the consumers in the opt-out pool until it covers the full pool of $\pi_W (1 - \gamma^*)$. Before it hits the corner, the interior choice is the first order condition in Equation (20) with $y_{j,in}^{GDPR} = 0$, which gives

$$y_{j,out}^{GDPR} = 1 - \frac{4cJ}{\bar{v}} (1 - \gamma^*).$$

If $1 - \frac{4cJ}{\bar{v}} (1 - \gamma^*) \geq \pi_W (1 - \gamma^*)$, which is equivalent to $\gamma^* \geq 1 - \left(\frac{4cJ}{\bar{v}} + \pi_W \right)^{-1}$, then $y_{j,out}^{GDPR} = \bar{y}_{out} = \pi_W (1 - \gamma^*)$ and $y_{j,in}^{GDPR}$ is given by the first order condition in Equation (19):

$$\left(\gamma^* - \frac{J}{\pi_W} y_{j,in}^{GDPR} \right) (1 - \bar{y}_{out} - y_{j,in}^{GDPR}) = \frac{c}{\bar{v}}.$$

This equation takes the advertising to the opt-out pool $y_{j,out}^{GDPR} = \bar{y}_{out}$ as given and solves for the advertising to the opt-in pool $y_{j,in}^{GDPR}$. Generically, we define $y_{in^*}(x)$ as the solution to

²²We also recognize that $\frac{c}{\bar{v}}$ is the minimum γ^* at which temptation goods firms advertises a positive amount to the opt-in pool. This value is recovered by recognizing at zero advertising to the opt-in pool (i.e., $y_{j,in}^{GDPR} = 0$), the first-order condition is $\bar{v}\gamma^* - c$, which is nonpositive if $\gamma^* \leq \frac{c}{\bar{v}}$. Notice, however, the temptation good firm also chooses zero advertising for the opt-out pool because $\gamma^* > \underline{\gamma}$, and the marginal revenue of the opt-in is always higher than that of the opt-out pool.

the following equation:

$$\left(\gamma^* - \frac{J}{\pi_W} y\right) (1 - x - y) = \frac{c}{\bar{v}},$$

which gives y the optimal amount of advertising to the opt-in pool for a given level of advertising x to the opt-out pool. This leads to

$$y^2 - \left(\frac{\pi_W}{J}\gamma^* + (1 - x)\right) y + \frac{\pi_W}{J}\gamma^* (1 - x) - \frac{\pi_W}{J} \frac{c}{\bar{v}} = 0.$$

As the larger root of this equation is larger than 1, we choose the smaller root:

$$y_{in^*}(x) = \frac{1}{2} \left((1 - x) + \frac{\pi_W}{J}\gamma^* \right) - \sqrt{\frac{1}{4} \left((1 - x) - \frac{\pi_W}{J}\gamma^* \right)^2 + \frac{\pi_W}{J} \frac{c}{\bar{v}}}. \quad (22)$$

Thus, in this case, $y_{j,in}^{GDPR} = y_{in^*}(\bar{y}_{out})$.

We now consider the case $\gamma^* > \underline{\gamma}$. In this case, the opt-in pool has priority. Before the marginal revenue of the opt-in pool drops down to that of the opt-out pool, we have an interior solution for $y_{j,in}^{GDPR} = y_{in^*}(0)$ with $y_{j,out}^{GDPR} = 0$.

These two marginal revenues will intersect at a unique level $y_{in^{**}}$ for $y_{j,in}^{GDPR}$, where

$$y_{in^{**}}(\gamma^*) = \frac{\pi_W}{J}\gamma^* - \left(\frac{\pi_W}{J}\right)^2 \frac{\frac{1}{4} - \left(\gamma^* - \frac{1}{2}\right)^2 1_{\{\gamma^* > \frac{1}{2}\}}}{(1 - \gamma^*) \pi_W},$$

which is positive whenever $\gamma^* \geq \underline{\gamma}$. We can further simplify

$$y_{in^{**}}(\gamma^*) = \begin{cases} \frac{\pi_W}{J} \left(\gamma^* - \frac{1}{4J} \frac{1}{1 - \gamma^*} \right) & \text{if } \underline{\gamma} \leq \gamma^* \leq \frac{1}{2} \\ \frac{\pi_W}{J} \gamma^* \left(1 - \frac{1}{J} \right) & \text{if } \gamma^* > \frac{1}{2} \end{cases}. \quad (23)$$

Note if $y_{in^*}(0)$ rises above $y_{in^{**}}(\gamma^*)$, it becomes profitable for the firm to target the opt-out pool together with the opt-in pool. In this situation, $y_{j,in}^{GDPR} = y_{in^{**}}(\gamma^*)$, then the first-order condition in Equation (19) determines the interior level of $y_{j,out}^{GDPR} = y_{out^*}(x)$ for a given level of $y_{j,in}^{GDPR} = x$ with

$$y_{out^*}(x) = \min \left\{ \max \left\{ 1 - \frac{c}{\bar{v}} \left(\gamma^* - \frac{x}{\pi_W/J} \right)^{-1} - x, 0 \right\}, \pi_W (1 - \gamma^*) \right\}. \quad (24)$$

In this expression, $y_{j,out}^{GDPR}$ is bounded from above by the size of the opt-out pool $\pi_W (1 - \gamma^*)$.

Consequently, substituting Equation (24) with Equation (23)

$$y_{out*}(y_{in**}(\gamma^*)) = \min \{ \max \{ z, 0 \}, \pi_W (1 - \gamma^*) \}. \quad (25)$$

where

$$z = \begin{cases} 1 - \frac{4Jc}{\bar{v}} (1 - \gamma^*) - \frac{\pi_W}{J} \left(\gamma^* - \frac{1}{4J} \frac{1}{1 - \gamma^*} \right) & \text{if } \underline{\gamma} \leq \gamma^* \leq \frac{1}{2} \\ 1 - \frac{Jc}{\gamma^* \bar{v}} - \frac{\pi_W \gamma^*}{J} \left(1 - \frac{1}{J} \right) & \text{if } \gamma^* > \frac{1}{2} \end{cases}. \quad (26)$$

If $y_{j,out}^{GDPR}$ is constrained at its upper bound $\bar{y}_{out} = \pi_W (1 - \gamma^*)$, then the first-order condition in Equation (19) gives an interior level of $y_{j,in}^{GDPR}$, with $y_{j,out}^{GDPR} = \bar{y}_{out}$. That is $y_{j,in}^{GDPR} = y_{in*}(\bar{y}_{out})$.

Taken together, if $\gamma^* \leq \underline{\gamma}$, the opt-out pool has priority:

$$y_{j,out}^{GDPR} = \min \left\{ 1 - \frac{4cJ}{\bar{v}} (1 - \gamma^*), \bar{y}_{out} \right\}, \quad (27)$$

and

$$y_{j,in}^{GDPR} = \begin{cases} 0 & \text{if } y_{j,out}^{GDPR} < \bar{y}_{out} \\ y_{in*}(\bar{y}_{out}) & \text{if } y_{j,out}^{GDPR} = \bar{y}_{out}. \end{cases} \quad (28)$$

If $\gamma^* > \underline{\gamma}$, the opt-in pool has priority:

$$y_{j,in}^{GDPR} = \begin{cases} y_{in*}(0) & \text{if } y_{in*}(0) < y_{in**}(\gamma^*) \\ y_{in**}(\gamma^*) & \text{if } y_{in*}(0) \geq y_{in**}(\gamma^*) \text{ and } y_{out*}(y_{in**}(\gamma^*)) < \bar{y}_{out} \\ y_{in*}(\bar{y}_{out}) & \text{if } y_{out*}(y_{in**}(\gamma^*)) \geq \bar{y}_{out} \end{cases}, \quad (29)$$

and

$$y_{j,out}^{GDPR} = \begin{cases} 0 & \text{if } y_{in*}(0) < y_{in**}(\gamma^*) \\ y_{out*}(y_{in**}(\gamma^*)) & \text{if } y_{in*}(0) \geq y_{in**}(\gamma^*) \text{ and } y_{out*}(y_{in**}(\gamma^*)) < \bar{y}_{out} \\ \bar{y}_{out} & \text{if } y_{out*}(y_{in**}(\gamma^*)) \geq \bar{y}_{out} \end{cases}. \quad (30)$$

Weak-willed customers: We first verify if other weak-willed customers follow the conjectured cutoff strategy with cutoff γ^* , it is optimal for a weak-willed consumer with

temptation γ_i to follow the same cutoff strategy. We then characterize the equilibrium cutoff γ^* .

Consider a weak-willed consumer with temptation index γ_i . Following Equation (7), his expected utility from opt-in is

$$U_{W,in}^{GDPR}(\gamma_i) = \frac{y_{n,in}^{GDPR} N}{\pi_S + \gamma^* \pi_W} \frac{\bar{u}}{8} + \frac{y_{j,in}^{GDPR}(\gamma_i) J}{\pi_W} (u_B - \gamma_i \bar{v}).$$

This expression shows that $U_{W,in}^{GDPR}$ increases with $y_{n,in}^{GDPR}$ but decreases with $y_{j,in}^{GDPR}(\gamma_i)$. Following Equation (8), his expected utility from opt-out is

$$U_{W,out}^{GDPR}(\gamma_i) = \frac{y_{n,out}^{GDPR}}{(1-\gamma^*)\pi_W} \frac{\bar{u}}{8} + \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W} u_B - \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W} \cdot \left[\max \left\{ \frac{1}{2}, \gamma^* \right\} \mathbf{1}_{\{\gamma_i > \max\{\frac{1}{2}, \gamma^*\}\}} + \gamma_i \mathbf{1}_{\{\gamma_i \leq \max\{\frac{1}{2}, \gamma^*\}\}} \right] \bar{v},$$

which increases with $y_{n,out}^{GDPR}$ and decreases with $y_{j,out}^{GDPR}$. Then,

$$\begin{aligned} V(\gamma_i) &= U_{W,in}^{GDPR}(\gamma_i) - U_{W,out}^{GDPR}(\gamma_i) \tag{31} \\ &= \left[\frac{y_{n,in}^{GDPR} N}{\pi_S + \gamma^* \pi_W} - \frac{y_{n,out}^{GDPR}}{(1-\gamma^*)\pi_W} \right] \frac{\bar{u}}{8} + \left(\frac{y_{j,in}^{GDPR}(\gamma_i)}{\pi_W/J} - \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W} \right) (u_B - \bar{v}\gamma_i) \\ &\quad - \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W} \bar{v} \left(\gamma_i - \max \left\{ \frac{1}{2}, \gamma^* \right\} \right) \mathbf{1}_{\{\gamma_i > \max\{\frac{1}{2}, \gamma^*\}\}} \end{aligned}$$

Note that $\frac{y_{n,in}^{GDPR} N}{\pi_S + \gamma^* \pi_W} \geq \frac{y_{n,out}^{GDPR}}{(1-\gamma^*)\pi_W}$ from our earlier analysis of firm n 's strategy. Therefore, whether $U_{W,in}^{GDPR}(\gamma_i) - U_{W,out}^{GDPR}(\gamma_i)$ crosses zero depends on the second and third terms. In the second term, whether $\frac{y_{j,in}^{GDPR}(\gamma_i)}{\pi_W/J} - \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W}$ is positive or not depends on whether γ^* is higher or lower than $\underline{\gamma}$.

We first show that in equilibrium γ^* cannot be lower than $\underline{\gamma}$. We use contradiction. Suppose that $\gamma^* < \underline{\gamma}$. Then, $\frac{y_{j,in}^{GDPR}(\gamma_i)}{\pi_W/J} \leq \frac{y_{j,out}^{GDPR}}{(1-\gamma^*)\pi_W}$ because the temptation good firms give higher priority to opt-out pool. It is then clear that $V(\gamma_i) > 0$ for γ_i slightly above γ^* , implying that this consumer would choose opt-in. This contradicts with γ^* being the equilibrium threshold so that consumers with γ_i above γ^* all choose opt-out.

For $\gamma^* \geq \underline{\gamma}$, define the (adjusted) net benefit to opt-in for the marginal weak-willed

consumer when she follows the conjectured cutoff strategy:

$$\begin{aligned}
C(\gamma^*) &\equiv \frac{1}{\bar{v}} (U_{W,in}^{GDPR}(\gamma^*) - U_{W,out}^{GDPR}(\gamma^*)) \\
&= \frac{\bar{u}}{8\bar{v}} \left(\frac{y_{n,in}^{GDPR} N}{\pi_S + \gamma^* \pi_W} - \frac{y_{n,out}^{GDPR}}{(1 - \gamma^*) \pi_W} \right) \\
&\quad + \left(\frac{u_B}{\bar{v}} - \gamma^* \right) \left(\frac{y_{j,in}^{GDPR}(\gamma^*)}{\pi_W/J} - \frac{y_{j,out}^{GDPR}}{(1 - \gamma^*) \pi_W} \right).
\end{aligned} \tag{32}$$

For γ^* to be the equilibrium cutoff, there are three possibilities:

$$\gamma^* \begin{cases} = \underline{\gamma} & \text{if } C(\underline{\gamma}) < 0 \\ \in (\underline{\gamma}, 1) & \text{if } C(\gamma^*) = 0 \\ = 1 & \text{if } C(1) > 0 \end{cases} . \tag{33}$$

Corner Solution for $\gamma^* = 1$: We consider this corner as a limiting case. There are two reasons why all consumers would choose opt-in.

First, the severity of temptation \bar{v} may be so high that temptation good firms will search the opt-out pool on the margin if all weak-willed consumers opt-in. Suppose a fraction ε of weak-willed consumers opt-out, i.e., $\gamma^* = 1 - \varepsilon$, then substituting for $y_{n,in}^{GDPR}$, Equation (32) reduces to

$$C(1 - \varepsilon) = \frac{\bar{u}}{8\bar{v}} \left(\min \left\{ \left(1 - 2\sqrt{\frac{c}{\bar{u}}} \right) \frac{N}{1 - \varepsilon \pi_W}, 1 \right\} - \frac{y_{n,out}^{GDPR}}{\varepsilon \pi_W} \right) + \left(\frac{u_B}{\bar{v}} + \varepsilon - 1 \right) \left(1 - \frac{y_{j,out}^{GDPR}}{\varepsilon \pi_W} \right).$$

Recognizing that unless normal good sellers can cover all consumers, they will eschew the opt-out pool ($y_{n,out}^{GDPR} = 0$) with a $\varepsilon \pi_W$ mass of consumers because it has lower expected revenue. Even though temptation firms give priority to the opt-in pool, a temptation good firm j may still cover the opt-out pool if $y_{in^*}(0)$ rises above $y_{in^{**}}(\gamma^* = 1 - \varepsilon)$, which is given by Equation (23). This condition $\varepsilon \rightarrow 0$ is equivalent to

$$\bar{v} \geq v_{**} \equiv \frac{c}{\left(1 - \frac{\pi_W}{J}\right) \frac{1}{J} + \frac{\pi_W}{J^3}}.$$

Second, the severity of temptation \bar{v} may be so low that all weak-willed consumers choose opt-in for the benefit of matching with normal good firms despite the cost of being targeted by temptation good firms. In this case, temptation good firms do not also advertise to the

opt-out pool ($\frac{y_{j,out}^{GDPR}}{\varepsilon\pi_W} = 0$) and $C(1 - \varepsilon)$ reduces to

$$C(1 - \varepsilon) = \frac{\bar{u}}{8\bar{v}} \min \left\{ \left(1 - 4\frac{c}{\bar{u}}\right) \frac{N}{1 - \varepsilon\pi_W}, 1 \right\} + \frac{u_B}{\bar{v}} + \varepsilon - 1.$$

Then $C(1 - \varepsilon)$ is positive as $\varepsilon \rightarrow 0$ if

$$\bar{v} < \frac{\bar{u}}{8} \min \left\{ \left(1 - \frac{4c}{\bar{u}}\right) N, 1 \right\} + u_B, \quad (34)$$

and again all weak-willed consumers opt-in. Similarly, if $\bar{v} < c$, then the temptation good firm never advertises to the opt-in pool because the marginal revenue \bar{v} is always less than the marginal cost, c . As such, all weak-willed consumers opt-in if

$$\bar{v} < \min c, \frac{\bar{u}}{8} \min \left\{ \left(1 - \frac{4c}{\bar{u}}\right) N, 1 \right\} + u_B \quad (35)$$

Note that the conditions $\bar{v} \leq v^*$ and Equation (35) may overlap. When this happens, there are two equilibria. In the equilibrium with $\gamma^* = 1$, all weak-willed consumers choose opt-in to maximize matching with normal good firms. In this case, the opt-out pool offers no protection for the severely tempted consumers. In the other equilibrium with $\gamma^* = \underline{\gamma}$, only a fraction of weak-willed consumers choose opt-in, and the opt-out pool provides substantial protection for the most tempted consumers. The complementarity in the consumers' opt-out decision contributes to the rise of multiple equilibria.

Interior Solution for $\gamma^* \in (\underline{\gamma}, 1)$: Note that $V(\gamma_i)$ in Equation (31) is monotonically decreasing with γ_i . Given that $V(\gamma^*) = C(\gamma^*) = 0$, consumers with $\gamma_i < \gamma^*$ want to opt-in and those with $\gamma_i > \gamma^*$ want to opt-out, confirming the optimality of the cutoff strategy for weak-willed consumers.

Given the non-linearity of $C(\gamma^*)$, there may be multiple values in $(\underline{\gamma}, 1)$ with $C(\gamma^*) = 0$, and consequently multiple equilibria. In this case, note that $\frac{y_{j,in}^{GDPR}(\gamma^*)}{\pi_W/J} = 1$ and $\frac{y_{j,out}^{GDPR}}{\pi_W(1-\gamma^*)} < 1$. The optimal advertising policy of firm j for the opt-in and opt-out pools is given by Equations (29) and (30). Substituting for $y_{n,in}^{GDPR}$ and $y_{n,out}^{GDPR}$ with Equations (15) and (16), we recognize

$$\begin{aligned} & \frac{y_{n,in}^{GDPR}N}{\pi_S + \gamma^*\pi_W} - \frac{y_{n,out}^{GDPR}}{\pi_W(1-\gamma^*)} \\ = & \min \left\{ \frac{1 - 4\frac{c}{\bar{u}}}{1 - \pi_W(1-\gamma^*)} N, 1 \right\} - \max \left\{ 1 - \frac{4N\frac{c}{\bar{u}}}{\pi_W(1-\gamma^*)}, 0 \right\}, \end{aligned}$$

and substituting this into Equation (32) gives

$$C(\gamma^*) = \frac{\bar{u}}{8\bar{v}} \left(\min \left\{ \frac{1 - 4\frac{c}{\bar{u}}}{1 - \pi_W(1 - \gamma^*)} N, 1 \right\} - \max \left\{ 1 - \frac{4N\frac{c}{\bar{u}}}{\pi_W(1 - \gamma^*)}, 0 \right\} \right) + \left(\frac{u_B}{\bar{v}} - \gamma^* \right) \left(1 - \frac{y_{j,out}^{GDPR}}{\pi_W(1 - \gamma^*)} \right), \quad (36)$$

where $y_{j,out}^{GDPR}$ is given by Equation (30).

The first term is continuous and positive on γ^* , and not equal to zero because $1 > 4\frac{c}{\bar{u}}$ by assumption. Whenever the max term is positive, the min term must be 1. The second term is continuous in γ^* because $\frac{y_{j,out}^{GDPR}}{\pi_W(1 - \gamma^*)} \in [0, 1]$ is (piece-wise) continuous in γ^* from Equation (30). Consequently, $C(\gamma^*)$ is continuous in γ^* on $[\underline{\gamma}, 1]$.

Thus, by the Intermediate Value Theorem, there exists a $\gamma^* \in (\underline{\gamma}, 1)$ such that $C(\gamma^*) = 0$, and an interior equilibrium exists. Notice

$$C(\underline{\gamma}) > \frac{\bar{u}}{8\bar{v}} \left(\min \left\{ \frac{1 - 4\frac{c}{\bar{u}}}{1 - \pi_W(1 - \underline{\gamma})} N, 1 \right\} - \max \left\{ 1 - \frac{4N\frac{c}{\bar{u}}}{\pi_W(1 - \underline{\gamma})}, 0 \right\} \right) + \frac{u_B}{\bar{v}} - \underline{\gamma}.$$

It is sufficient that $c > \frac{\bar{u}}{N}\pi_W\frac{1-\underline{\gamma}}{4}$ (i.e., normal good firms ignore the opt-out pool and advertise only to the opt-in pool) and

$$\bar{v} < v_{***} \equiv \frac{u_B}{\underline{\gamma}} + \frac{\bar{u}}{\underline{\gamma}} \min \left\{ \frac{N - \pi_W(1 - \underline{\gamma})}{1 - \pi_W(1 - \underline{\gamma})}, 1 \right\}, \quad (37)$$

for $C(\underline{\gamma}) > 0$. Consequently, it is sufficient that $c > \frac{\bar{u}}{N}\pi_W\frac{1-\underline{\gamma}}{4}$ and $\bar{v} < v_{***}$ to ensure there is an interior equilibrium.

When, in addition, $\bar{v} > v_{**}$, then there are multiple equilibrium in which full data-sharing is an equilibrium.

A.6 Proof of Proposition 7

We first compare social welfare under three data sharing schemes: no data sharing, full data sharing, and the GDPR. Under a specific data-sharing scheme, social welfare is determined by the aggregate utility of strong- and weak-willed consumers over the consumption goods, as indicated by Equation (3). This is based on the assumptions that the marginal cost of goods production is zero, and the prices of goods and advertising costs are zero-sum transfers within the population. As normal good firm n cannot price discriminate against its customers due to the consumers' random utility for normal goods, it always charges a price of $\bar{u}/2$ for

its good. Consequently, only half of the intended consumers with random utility above $\bar{u}/2$ consume the good. Thus, consumers' net utility gain from good n is $\frac{3}{8}\bar{u}\rho_n$, where ρ_n is the measure of strong-willed and weak-willed consumers receiving firm n 's advertising. For a temptation good j , weak-willed consumers who purchase the good (with a measure of ρ_j) experience a negative utility of $u_B < 0$. Meanwhile, those who receive advertising from firm j but resist the temptation (marked in a set S_j) suffer a mental cost of $u_B - \gamma_i\bar{v}$. Note that ρ_n , ρ_j , and S_j are determined by the firms' advertising and pricing strategies under each of the data sharing schemes.

Taken together, the social welfare is

$$W = \frac{3}{8} \sum_{n=1}^N \bar{u}\rho_n + \sum_{j=N+1}^{N+J} \left[u_B\rho_j + \int_{i \in S_j} (u_B - \gamma_i\bar{v}) dG(\gamma_i) \right]. \quad (38)$$

Across the data sharing schemes, the key trade-off is between the first term (the benefit from normal goods) and the second and third terms (the cost from temptation goods). Note that these terms only account for the consumers' utility from the normal and temptation goods without including the prices they pay for the goods, which are transfers within the population.

No data sharing: From the proof of Proposition 5, the social welfare is

$$W^{NS} = \frac{3}{8}\bar{u}(\pi_S + \pi_W)y_n^{NS} + \pi_W y_j^{NS} \left(u_B - \frac{\bar{v}}{8} \right) = \frac{3}{8}\bar{u}y_n^{NS} + \pi_W y_j^{NS} \left(u_B - \frac{\bar{v}}{8} \right),$$

because $\pi_S + \pi_W = 1$.

Full data sharing: From the proof of Proposition 5, the social welfare is

$$W^{FS} = \frac{3}{8}\bar{u}N y_n^{FS} + J y_j^{FS} u_B.$$

We have from Proposition 5 that no data sharing dominates full data sharing if u_B is sufficiently negative, $W^{NS} \geq W^{FS}$, and is dominated otherwise, $W^{NS} < W^{FS}$.

GDPR: From a social welfare perspective, we have

$$W^{GDPR} = (N y_{n,in}^{GDPR} + y_{n,out}^{GDPR}) \frac{3}{8}\bar{u} + (J y_{j,in}^{GDPR} + y_{j,out}^{GDPR}) u_B - y_{j,out}^{GDPR} \left(\frac{1}{8} - \frac{\gamma^{*2}}{2} \right) \bar{v} \mathbf{1}_{\gamma^* < \frac{1}{2}},$$

where the last term reflects that the least tempted weak-willed in the opt-out pool suffer a temptation cost if $\gamma^* < \frac{1}{2}$.

We now compare the GDPR to no data sharing:

$$\begin{aligned} W^{GDPR} - W^{NS} &= (Ny_{n,in}^{GDPR} + y_{n,out}^{GDPR} - y_n^{NS}) \frac{3}{8} \bar{u} + (Jy_{j,in}^{GDPR} + y_{j,out}^{GDPR} - y_j^{NS}) u_B \\ &\quad + \pi_W y_j^{NS} \frac{\bar{v}}{8} - y_{j,out}^{GDPR} \left(\frac{1}{8} - \frac{\gamma^{*2}}{2} \right) \bar{v} \mathbf{1}_{\gamma^* < \frac{1}{2}}, \end{aligned}$$

where y_n^{NS} and y_j^{NS} are independent of u_B . The first term in $W^{GDPR} - W^{NS}$ is positive, representing the improved matching with normal good firms under opt-in/opt-out, while the second is negative, reflecting the increased exposure of weak-willed consumers to temptation goods firms.

Notice when $u_B = 0$, it must be the case $W^{GDPR} > W^{NS}$ because of the improved matching with normal goods firms. When $u_B < 0$ however, the most-tempted weak-willed consumers suffer from lack of camouflage because not only all strong-willed, but also the more-mildly tempted weak-willed, consumers opt-in. Because the social benefit of GDPR from increased matching with normal goods firms is bounded from above by $(Ny_{n,in}^{GDPR} - y_n^{NS}) \frac{3}{8} \bar{u}$, it follows for sufficiently negative u_B that $W^{GDPR} < W^{NS}$. Since the objectives are continuous, there exist critical values of u_B , u_{B**} , such that $W^{GDPR} < W^{NS}$ when $u_B \leq u_{B**}$.

We now compare the GDPR with the full data sharing. The difference in the social welfare is given by

$$\begin{aligned} W^{GDPR} - W^{FS} &= (Ny_{n,in}^{GDPR} + y_{n,out}^{GDPR} - Ny_n^{FS}) \frac{3}{8} \bar{u} + (Jy_{j,in}^{GDPR} + y_{j,out}^{GDPR} - Jy_{j,out}^{FS}) u_B \\ &\quad - y_{j,out}^{GDPR} \left(\frac{1}{8} - \frac{\gamma^{*2}}{2} \right) \bar{v} \mathbf{1}_{\gamma^* < \frac{1}{2}}. \end{aligned}$$

Note that under full data sharing, normal goods firms have higher advertising efficiency and therefore are able to better cover their intended consumers, that is, the first term is negative. It is further clear that total advertising by temptation goods firms under opt-in/opt-out is less than that under full data sharing, $Jy_{j,in}^{GDPR} + y_{j,out}^{GDPR} - Jy_{j,out}^{FS} < 0$. Because temptation goods firms are less efficient at targeting the most-tempted customers, the coefficient of u_B in the last term is negative, i.e., the second term is positive.

Consequently, there may exist a critical u_{B*} such that $W^{GDPR} > W^{FS}$ if $u_B \leq u_{B***}$ (and $W^{GDPR} < W^{FS}$ otherwise).

Ranking the three schemes: Suppose u_B is sufficiently severe ($u_B < \min \{u_{B*}, u_{B***}\}$), then $W^{NS} > W^{GDPR} > W^{FS}$, and no data sharing delivers the highest social welfare.

Further, it is sufficient, although not necessary, for u_B to be in an intermediate range ($u_B < u_{B^{***}}$ and $u_B > u_{B^{**}}$), for $W^{GDPR} > W^{FS}$, W^{NS} , and GDPR delivers the highest social welfare.

Comparing the welfare gap: We next compare the welfare gap relative to no data sharing. From Proposition 5, the welfare gap is higher under full data sharing than no data sharing.

We now establish that the welfare gap is also higher under GDPR than no data-sharing. First, notice the welfare gap from Equation (4) is divided into two pieces the difference in utility from normal goods Δ_{GDPR}^n and the drag on weak-willed consumer welfare from temptation goods Δ_{GDPR}^j . Because there are fewer weak-willed than strong-willed consumers, and some weak-willed consumers opt-out, strong-willed consumers differentially benefit more from improved access to normal goods by opting-in. As such, the first term in the welfare gap $\Delta_{GDPR}^n > 0$ is higher under GDPR than no data sharing, i.e., $\Delta_{GDPR}^n > \Delta_{NS}^n$. In addition, because temptation goods firms can better target weak-willed consumers when a subset opts-in, $\Delta_{GDPR}^j < 0$ is also more negative under GDPR (i.e., the profits of temptation goods firms are higher), or $\Delta_{GDPR}^j > \Delta_{NS}^j$. Consequently:

$$\Delta_{GDPR} = \Delta_{GDPR}^n + \Delta_{GDPR}^j > \Delta_{NS}^n + \Delta_{NS}^j = \Delta_{NS}.$$

As such, the welfare gap is smallest under no data sharing.