Abstract

We develop a model of neighborhood choice to analyze information aggregation and learning in housing markets. In the presence of pervasive informational frictions, housing prices serve as important signals to households and capital producers about the economic strength of a neighborhood. Through this learning channel, noise from housing market supply and demand shocks can propagate from housing prices to the local economy, distorting not only migration into the neighborhood, but also the supply of capital and labor. We confirm several key predictions of our model for housing price volatility and new construction using data from the recent U.S. housing cycle.
Widespread optimism is well-recognized as a key driver of the recent U.S. housing bubble, e.g., Cheng, Raina and Xiong (2014), Kaplan, Mitman, and Violante (2017), and Soo (2018). The literature often attributes such optimism during housing booms to home buyers’ extrapolative expectations, e.g., Glaeser, Gyourko and Saiz (2008) and Glaeser and Nathanson (2017). Intuitively, by amplifying housing price fluctuations, extrapolation makes housing cycles monotonic with respect to the supply elasticity of land.\(^1\) This prediction, however, does not fully capture the cross-section of the recent U.S. housing cycle. Many researchers, including Glaeser (2013), Davido¤ (2013), and Nathanson and Zwick (2018), have noted that the housing price boom and bust were most pronounced in areas that were not particularly constrained by the supply of land, including Las Vegas and Phoenix. We systematically document that this non–monotonic pattern with respect to supply elasticity was more ubiquitous during the recent U.S. housing bubble than previously appreciated—in not only the magnitudes of the housing price boom and bust but also in new construction. These non-monotonic patterns suggest the need for a new economic mechanism for beliefs to interact with housing cycles beyond extrapolation.

In this paper, we address this challenge by developing a model to analyze how informational frictions affect the learning and beliefs of households and capital producers about a neighborhood, which, in turn, drive both local housing market dynamics and investment decisions. Our model extends the problem of coordination with dispersed information to investigate its role in amplifying the agglomeration effects that underpin the formation of neighborhoods and cities. Our model is able to generate rich non-monotonic patterns in housing cycles with respect to supply elasticity, as well as the degree of local industry diversity, another dimension in which counties differ.\(^2\)

Our model features a continuum of households, each of which has the choice of whether to move into an open neighborhood, which can be viewed as a county, by buying a house. Each

\(^1\)Glaeser, Gyourko and Saiz (2008) caveats that, while housing price appreciation and new construction during a housing boom are monotonically increasing and decreasing with supply elasticity, respectively, the subsequent housing bust can be U-shaped as a result of the overhang of excessive housing constructed during the boom. That is, excessive new construction during the boom may cause areas with intermediate elasticity to suffer more pronounced busts than more inelastic areas with less construction.

household has a Cobb-Douglas utility function over its consumption of its own good and its aggregate consumption of the goods produced by other households in the neighborhood. This complementarity in households’ consumption conveniently captures local industry diversity and motivates each household to learn about the unobservable economic strength of the neighborhood, which determines the common productivity of all households and leads to complementarity in their housing demand. As opposed to regional specialization, in which similar firms benefit from competing for a similar pool of customers and workers, industry diversity reflects that dissimilar firms benefit from each other’s goods, services, and knowledge. Each household requires both labor, which it supplies, and local capital to produce its good according to a Cobb-Douglas production function. Since the price of capital depends on its marginal product across households in the neighborhood, competitive capital producers must also form expectations about the neighborhood’s economic strength when determining how much local capital to develop.

Although previously unexplored in the housing literature, it is intuitive that local housing markets provide a useful platform for aggregating private information about the economic strength of a neighborhood. The traded housing price reflects the net effect of demand and supply-side factors, in a similar spirit to the classic models of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets. Different from the linear equilibrium in these models, our setting features an important neighborhood selection effect, through which only households with private signals above a certain equilibrium cutoff choose to live in the neighborhood. In addition, the complementarity between labor and capital in household production gives rise to a feedback effect in which households must forecast each capital producer’s investment decisions when deciding whether to live in the neighborhood, and capital producers must forecast each household’s housing purchase decision when deciding how much capital to supply. As such, this neighborhood selection effect along with production makes our model inherently nonlinear, which poses a substantial challenge to maintaining tractability in both household learning and information aggregation with dispersed information. Nevertheless, we are able to derive the equilibrium analytically, building on and extending the cutoff equilibrium framework developed by Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2015, 2017) for asset markets.

There are two key features that contribute to this tractability. First, despite the equilibrium housing price being a nonlinear function, its information content about the neigh-
borhood strength can be summarized by a linear sufficient statistic, which keeps households’
learning from the housing price tractable. Second, despite each household’s housing demand
being nonlinear, the Law of Large Numbers allows us to aggregate their housing demand,
and to derive a cutoff equilibrium for the housing market. In our setting, each household
possesses a private signal regarding the neighborhood’s common productivity. By aggregat-
ing the households’ housing demand, the housing price aggregates their private signals. The
presence of unobservable supply shocks, however, prevents the housing price from perfectly
revealing the neighborhood’s strength, and acts as a source of informational noise in the
housing price.

Our model allows us to analyze how informational frictions affect not only the housing
price, but also each household’s neighborhood, labor and production decisions. These, in
turn, determine each household’s demand for housing and capital. Since the housing price
serves as a signal about the neighborhood’s strength, it plays a key role in determining
agents’ expectations. Through this learning channel, noise in the housing market, originating
from either the demand or supply side, impacts the housing price and the local economy.
Noise that pushes up housing prices can induce more households to enter the neighborhood
through learning, since a higher price signals a stronger neighborhood. This extrapolative-
like behavior anchoring on learning leads to not only a more pronounced housing cycle but
also an oversupply of new housing and local capital, consistent with the empirical findings
of Gao, Sockin, and Xiong (2019).

Our analysis illustrates how the transmission of noise in housing markets to real estate
and production outcomes varies across different neighborhoods by the elasticity of their
housing supply. With respect to supply elasticity, the distortion to housing prices induced
by noise through the learning channel is hump-shaped. At one extreme, when housing supply
is infinitely inelastic, the housing price is fully determined by housing demand, and perfectly
reveals the strength of the neighborhood; at the other extreme, when housing supply is
perfectly elastic, housing prices are fully determined by housing supply, and are therefore
not affected by households’ expectations. At both extremes, learning does not distort the
housing price. As a result, the noise effect on housing prices from informational frictions is
strongest at intermediate supply elasticities. This insight helps to explain the aforementioned
hump-shaped patterns in the magnitudes of the housing price boom and bust and housing
construction during the recent U.S. housing cycle with respect to supply elasticity.
Our analysis also examines this transmission across the degree of households’ consumption complementarity, which we view as industry diversity. Our analysis shows that distortionary effects in the housing market from learning tend to increase with complementarity, as greater complementarity makes learning about the neighborhood’s strength a more important part of each household’s decisions. In contrast, complementarity mitigates the distortions from learning to the market for capital, as greater coordination by households also lowers the average marginal product of capital for a given strength of the neighborhood. As a result of these two effects, our model also predicts a monotonically increasing pattern in the magnitudes of housing price boom and bust and a hump-shaped pattern in housing construction - with respect to the degree of complementarity. Interestingly, by sorting areas by the degree of complementarity of their industries empirically, we are able to confirm these patterns in the data.

By featuring a tractable cutoff equilibrium framework, our work contributes to a growing literature that analyzes information aggregation in nonlinear settings. Goldstein, Ozdenoren, and Yuan (2013) investigate the feedback to the investment decisions of a single firm when managers, but not investors, learn from prices. Albagli, Hellwig, and Tsyvinski (2015, 2017) focus on the role of asymmetry in security payoffs in distorting asset prices and firm investment incentives when future shareholders learn from prices to determine their valuations. These models commonly employ risk-neutral agents, normally distributed asset fundamentals, and position limits to deliver tractable nonlinear equilibria. In contrast, we focus on the feedback induced by learning from housing prices to household neighborhood choice and labor decisions in the presence of consumption complementarity and goods trading between households. We further analyze the spillover of this feedback to the investment decisions of capital producers. By showing that the cutoff equilibrium framework can be adopted to analyze learning effects in this richer setting, our model substantially expands the scope of this framework to a general equilibrium real business cycle setting. In this regard, our model adds to the growing literature, e.g., Bond, Edmans and Goldstein (2012), on the real effects of learning from trading prices.

While long appreciated as important explanation for housing market behavior, such as in Garmaise and Moskowitz (2004), Kurlat and Stroebel (2014), Favara and Song (2014), and Bailey et al. (2017), informational frictions have yet to be applied to understanding the recent U.S. housing cycle and its real effects. The literature has instead focused on other
causes ranging from credit expansion and fraudulent lending practices to speculation and optimistic, often extrapolative, expectations. By anchoring household expectations to local economic conditions, our theory provides guidance as to where optimism and overreaction had the most pronounced impact on housing and local economic outcomes during the boom, and offers novel empirical predictions on non-monotonic patterns in housing cycles and new construction with respect to supply elasticity and the degree of complementarity. In addition, our mechanism can rationalize the synchronized boom and bust cycles in commercial real estate markets, in which prices and new construction rose across the U.S. despite the bubble in housing (Gyourko (2009a) and Levitin and Wachter (2013)). By impacting the demand curve for housing, informational frictions complement the credit expansion and fraud channels and, by facilitating heterogeneous beliefs, can give rise to speculative demand.

Our model adds to the literature on the theoretical modeling of housing cycles. Burnside, Eichenbaum, and Rebelo (2016) offer a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Nathanson and Zwick (2018) study the hoarding of land by home builders with heterogeneous beliefs in intermediate elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Piazzesi and Schneider (2009) investigate how a small population of optimists can inflate housing prices by driving transaction volume. Glaeser and Nathanson (2017) presents a model of biased learning in housing markets in which the incorrect inference by home buyers gives rise to correlated errors in housing demand forecasts over time, which, in turn, generate excess volatility, momentum, and mean-reversion in housing prices. Guren (2016) develops a model of housing price momentum, building on the incentive of individual sellers not to set a unilaterally high or low list price because the demand curve they face is concave in the relative price. In contrast to these models, informational frictions in our framework anchor on the interaction between the demand and supply sides of the housing market, and feed back to both housing prices and real outcomes. This key feature is also distinct from the amplification of price volatility induced by dispersed information and short-sale constraints featured in Favara and Song (2014).

In addition, there are extensive studies in the housing literature highlighting the roles

3For credit expansion, see, for instance, Mian and Sufi (2009, 2011) and Albanesi et al. (2017). For fraudulent lending practices, see Keys et al. (2009) and Griffin and Maturana (2015). For speculation, see Nathanson and Zwick (2018), DeFusco, Nathanson, and Zwick (2017), and Gao, Sockin and Xiong (2019). For extrapolative expectations, see Case and Shiller (2003), Glaeser, Gyourko, and Saiz (2008), Piazzesi and Schneider (2009), and Glaeser and Nathanson (2017).
played by both demand-side and supply-side factors in driving housing cycles. On the de-
mand side, Himmelberg, Mayer, and Sinai (2005) focus on interest rates, Poterba, Weil, and
Shiller (1991) on tax changes, Mian and Sufi (2009) on credit expansion, and Chinco and
Mayer (2015), DeFusco, Nathanson, and Zwick (2017) and Gao, Sockin and Xiong (2019)
emphasizes the elasticity of housing supply as a key force in mitigating housing bubbles.4
By introducing informational frictions, our analysis shows that supply-side and demand-side
factors are not mutually independent. Supply shocks can affect housing and investment
demand by acting as informational noise in learning, and influence households’ and capital
producers’ expectations of the strength of a neighborhood.

1 Several Stylized Facts

In this section, we present several stylized facts regarding the relation between supply elas-
ticity and the housing dynamics experienced by different counties during the recent U.S.
housing cycle. The literature, for instance in Malpezzi and Wachter (2005) and Glaeser,
Gyourko, and Saiz (2008), has emphasized supply elasticity as a key factor in explaining
certain features of housing cycles, such as housing price volatility. The national U.S. housing
market underwent a significant boom and bust cycle in the 2000s with the national home
price index increasing over 60 percent from 2000 to 2006, and then falling back to its 2000
level by 2010. Many factors, such as the Clinton-era initiatives to broaden home ownership,
the low interest rate environment of the late 1990s and early 2000s, the inflow of foreign
capital, and the increase in securitization and sub-prime lending, contributed to the initial
housing boom. While a well-known phenomenon at the time, the magnitude of the hous-
ing cycle experienced in the cross-section of U.S. counties reflected idiosyncratic uncertainty
about their underlying fundamentals, which is the focus of our analysis.5 In what follows,
we analyze not only the housing price cycle, but also new housing construction. We design-
nate the boom period of the recent U.S. housing cycle as 2001-2006, and the bust period as

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4 Haughwout et al. (2013) provide a detailed account of the supply side of housing during the U.S. housing
cycle in the 2000s, and Gyourko (2009b) systematically reviews the literature on housing supply.

5 The regional uncertainty introduced by this national phenomenon is absent from the local boom and
bust episodes throughout the 1970s and 1980s. While there are other national housing cycles in history, such
as in the roaring 20’s, data limitations restrict our attention to the most recent U.S. housing cycle.
Our county-level house price data come from the Case-Shiller home price indices, which are constructed from repeat home sales. There are 420 counties in 46 states with a large enough number of repeat home sales to compute the Case-Shiller home price indices. For housing supply elasticity, we employ the commonly used elasticity measure constructed by Saiz (2010). This elasticity measure focuses on geographic constraints by defining undevelopable land for construction as terrain with a slope of 15 degrees or more and areas lost to bodies of water including seas, lakes, and wetlands. This measure provides an exogenous measure of supply elasticity, with a higher value if an area is more geographically restricted. Saiz’s measure is available for 269 Metropolitan Statistical Areas (MSAs). By matching counties with MSAs, our sample includes 326 counties for which we have data on both house prices and supply elasticity available from 2000 to 2010. Though our sample covers only 11 percent of the counties in the U.S., they represent 53 percent of the U.S. population and 57 percent of the housing trading volume in 2000.

To measure the supply-side activity in local U.S. housing markets, we use building permits from the U.S. Census Bureau, which conducts a survey in permit-issuing places all over the U.S. Compared with other construction-related measures, including housing starts and housing completions, building permits have detailed county-level information. In addition, building permits are issued right before housing starts and therefore can predict price trends in a timely manner.7 We measure new housing supply during the boom period by the building permits issued in 2001-2006 relative to the existing housing units in 2000.

The first two panels of Figure 1 provide scatter plots of the housing price expansion and contraction experienced by each county during the housing boom and bust periods, respectively. To conveniently summarize the data, we include a spline line to fit each of the scatter plots (the solid line in the plots), together with 95% confidence interval (the shaded area around the spline line). These spline lines clearly indicate that the housing cycle was non-monotonic with respect to supply elasticity—a hump-shaped pattern for the housing

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6See Gao, Sockin and Xiong (2019) for another study that uses a similar dating convention for the U.S. housing cycle in the 2000s.

7Authorization to start is a largely irreversible process, with housing starts being only 2.5% lower than building permits at the aggregate level according to https://www.census.gov/construction/nrc/nrcrelationships.html, the website of the Census Bureau. Moreover, the delay between authorization and housing start is relatively short, on average less than one month, according to https://www.census.gov/construction/nrc/lengthoftime.html. These facts suggest that building permits are an appropriate measure of new housing supply.
price appreciation during the boom and a U-shaped pattern for the price drop during the bust. In particular, the cycle was most pronounced for counties with intermediate, rather than the lowest, supply elasticities.

One may be concerned that this non-monotonicity in the housing cycle across supply elasticity is driven by the so-called "sand states" (Arizona, California, Florida, and Nevada). These four states experienced exceptional housing price booms and busts and, as several scholars including Davidoff (2013) and Nathanson and Zwick (2018) have noted, were characterized by peculiar speculative activities, such as land hoarding by real estate developers. In the scatter plots provided by Figure 1, we separately mark the counties in the sand states by "+" and provide a separate spline line (the dashed line in the plots) for observations excluding the sand-state counties. Indeed, the counties in the sand states experienced rel-
atively more pronounced price appreciations during the boom and more severe price drops during the bust. Despite excluding these counties, however, the hump-shaped pattern for the price appreciation during the boom and the U-shaped pattern for the price drop during the bust remain significant, albeit with more attenuated magnitudes. The non-monotonic patterns in the housing price boom and bust with respect to supply elasticity, therefore, represent robust phenomena beyond the sand states.

In addition to the housing price cycle, the third panel of Figure 1 provides a scatter plot of housing construction during the boom period, measured by new housing permits, with respect to supply elasticity. There is noticeably also a hump-shaped pattern with respect to supply elasticity, with counties in the intermediate elasticity range having the most new construction, instead of areas with the most elastic housing supply. This hump-shaped pattern is highly significant and robust to excluding the counties from the sand states. This surprising pattern in new construction has received little attention in the literature, and raises a fundamental question regarding the economic mechanism that drives housing market activities during a housing cycle beyond the price boom and bust.

Taken together, although common wisdom posits that supply elasticity attenuates housing cycles, we do not observe monotonic patterns across supply elasticity during the recent U.S. housing cycle in either the magnitude of the housing price boom and bust or in new construction. Instead, our analysis uncovers that counties with supply elasticities in an intermediate range experienced not only the most dramatic price cycles, but also the most new construction. Existing models of housing cycles have difficulty explaining these patterns. For example, Glaeser, Gyourko, and Saiz (2008) shows that, in the presence of housing supply constraints and extrapolative home buyer expectations, the overhang of housing supply developed during the boom may cause areas with intermediate supply elasticities to suffer the most dramatic housing price drops during the subsequent bust. Their analysis, however, shows that housing price appreciation is decreasing while new construction is increasing across supply elasticity during the boom when the boom period is synchronized in the cross-section. Furthermore, while Nathanson and Zwick (2018) identify land speculation by real estate developers as an important mechanism driving the recent housing boom in intermediate elastic areas such as Las Vegas, their analysis does not provide a systematic theory for the full spectrum of housing cycles experienced across areas with different supply elasticities. In addition, the hoarding of land by optimistic developers in intermediate elastic areas,
while exacerbating the housing price boom, has ambiguous implications about whether new construction is also the most pronounced in those counties.

2 The Model

The model has two periods $t \in \{1, 2\}$. There are three types of agents in the economy: households looking to buy homes in a neighborhood or elsewhere, home builders, and capital producers. Suppose that the neighborhood is new and all households purchase houses from home builders in a centralized market at $t = 1$ after choosing whether to live in the neighborhood. Households choose their labor supply and demand for capital, such as machines and office space, to complete production, and consume consumption goods at $t = 2$. Our intention is to capture the decision of a generation of home owners to move into a neighborhood, and we view the two periods as representing a long length of time in which they live together and share amenities, as well as exchange their goods and services.

2.1 Households

We consider a pool of households, indexed by $i \in [0, 1]$, each of which can choose to live in a neighborhood or elsewhere. We can divide the unit interval into the partition $\{N, O\}$, with $N \cap O = \emptyset$ and $N \cup O = [0, 1]$. Let $H_i = 1$ if household $i$ chooses to live in the neighborhood, i.e., $i \in N$, and $H_i = 0$ if it chooses to live elsewhere.\(^8\) If household $i$ at $t = 1$ chooses to live in the neighborhood, it must purchase one house at price $P$. This reflects, in part, that housing is an indivisible asset and a discrete purchase, consistent with the insights of Piazzesi and Schneider (2009).

To incorporate complementarity in households’ housing demands, we adopt a particular structure for their goods consumption and trading. Each household in the neighborhood produces a distinct good from the other households. Household $i$ has a Cobb-Douglas utility function over consumption of its own good $C_i(i)$ and its consumption of the goods produced by all other households in the neighborhood $\{C_j(i)\}_{j \in N}$:

$$U\left(\{C_j(i)\}_{j \in N} ; N\right) = \left(\frac{C_i(i)}{1 - \eta_c}\right)^{1 - \eta_c} \left(\frac{\int_{N \setminus i} C_j(i) \, dj}{\eta_c}\right)^{\eta_c}. \quad (1)$$

\(^8\)See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.
The parameter $\eta_c \in (0, 1)$ measures the weights of different consumption components in the utility function. A higher $\eta_c$ indicates a stronger complementarity between household $i$’s consumption of its own good and its consumption of the composite good produced by the other households in the neighborhood. One can view this complementarity as capturing agglomeration and spillover effects from households and firms locating near each other, or as reflecting that households and firms require each other’s intermediate goods and services as inputs to their own production.\footnote{Similar specifications of this utility function are employed, for instance, in Dixit and Stiglitz (1977) and Long and Plosser (1987) to give rise to input and output linkages in sectoral production. One can view \( \left( \frac{1}{1-\eta_c} \int_{\mathcal{N}/i} C_j(i) \, d\nu_j \right)^{1-\eta_c} \) as a final good produced by household $i$ given intermediate goods \{\( C_j(i) \)\}$_{j \in \mathcal{N}}$.} As we will discuss later, this utility specification implies that each household cares about the strength of the neighborhood, i.e., the productivity of other households in the neighborhood. This assumption leads to strategic complementarity in households’ housing demands, an important feature emphasized by the empirical literature, such as in Ioannides and Zabel (2003).\footnote{While our model builds on complementarity in household consumption, other types of social interactions between households in a neighborhood may also lead to complementarity in their housing demand, as discussed in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2003).}

The production function of household $i$ is also Cobb-Douglas $e^{A_i K_i^{\alpha_l}}$, where $A_i$ is its productivity, $l_i$ is the household’s labor choice, and $K_i$ is its choice of capital with a share of $\alpha \in (0, 1)$ in the production function. We broadly interpret capital as both public and private investment in the neighborhood, which can include office, machines, computers, and other equipment and infrastructure households can use for their productive activities.\footnote{In the case that $K$ is a public good, its price can be interpreted as the tax a local government that faces a balanced budget can raise to offset the cost of construction. Our model then has implications for how housing markets impact the fiscal policy of local governments.}

As we describe later, households buy capital from capital producers. When households are more productive in the neighborhood, the marginal productivity of capital is higher, and consequently capital producers are able to sell more capital at higher prices. Introducing capital allows us to discuss how learning affects the price and supply of not only residential housing, but also of local investment in the neighborhood.

Household $i$’s productivity $A_i$ is comprised of a component $A$, common to all households in the neighborhood, and an idiosyncratic component $\varepsilon_i$:

$$A_i = A + \varepsilon_i,$$

where $A \sim \mathcal{N}(\bar{A}, \tau_A^{-1})$ and $\varepsilon_i \sim \mathcal{N}(0, \tau_\varepsilon^{-1})$ are both normally distributed and independent.
of each other. Furthermore, we assume that \( \int \varepsilon_i d\Phi(\varepsilon_i) = 0 \) by the Strong Law of Large Numbers. The common productivity, \( A \), represents the strength of the neighborhood, as a higher \( A \) implies a more productive neighborhood. As \( A \) determines the households’ aggregate demand for housing, it also represents the demand-side fundamental.

As a result of realistic informational frictions, \( A \) is not observable to households at \( t = 1 \) when they need to make the decision of whether to live in the neighborhood. Instead, each household observes its own productivity \( A_i \), after examining what it can do if it chooses to live in the neighborhood. Intuitively, \( A_i \) combines the strength of the neighborhood \( A \) and the household’s own attribute \( \varepsilon_i \). Thus, \( A_i \) also serves as a noisy private signal about \( A \) at \( t = 1 \), as the household cannot fully separate its own attribute from the opportunity provided by the neighborhood. The parameter \( \tau_\varepsilon \) governs both the household diversity in the neighborhood and the precision of this private signal. As \( \tau_\varepsilon \to \infty \), the households’ signals become infinitely precise and the informational frictions about \( A \) vanish. Households care about the strength of the neighborhood because of complementarity in their demand for consumption. While a household may have a fairly good understanding of its own productivity when moving into a neighborhood, complementarity in consumption demand motivates it to pay attention to housing prices to learn about the average level of productivity \( A \) for the neighborhood.

We start with each household’s problem at \( t = 2 \) and then go backward to describe its problem at \( t = 1 \). At \( t = 2 \), we assume that \( A \) is revealed to all agents. Furthermore, we assume that each household experiences a disutility for labor \( \frac{l^{1+\psi}}{1+\psi} \), and that a household in the neighborhood (i.e., \( i \in \mathcal{N} \)) maximizes its utility at \( t = 2 \) by choosing labor \( l_i \), capital \( K_i \), and its consumption demand \( \{C_j(i)\}_{j \in \mathcal{N}} \):

\[
U_i = \max_{\{C_j(i)\}_{j \in \mathcal{N}, i, K_i}} U \left( \{C_j(i)\}_{j \in \mathcal{N}}; \mathcal{N} \right) - \frac{l_i^{1+\psi}}{1+\psi} \tag{2}
\]

such that

\[
p_i C_i(i) + \int_{\mathcal{N} \setminus i} p_j C_j(i) \, dj + RK_i = p_i \varepsilon A K_i^{\alpha_1} l_i^{1-\alpha_1},
\]

where \( p_i \) is the price of the good it produces and \( R \) is the unit price of capital. Households behave competitively and take the prices of their goods as given.

At \( t = 1 \), each household needs to decide whether to live in the neighborhood. In addition to their private signals, all households and capital producers observe a noisy public signal \( Q \) about the strength of the neighborhood \( A \):

\[
Q = A + \tau_Q^{-1/2} \varepsilon Q,
\]
where \( \varepsilon_Q \sim \mathcal{N}(0, 1) \) is independent of all other shocks. As \( \tau_Q \) becomes arbitrarily large, \( A \) becomes common knowledge to all agents. This public signal could, for instance, be news reports or published statistics on local economic conditions.

In addition to the utility flow \( U_i \) at \( t = 2 \) from goods consumption and labor disutility, we assume that households have quasi-linear expected utility at \( t = 1 \), and, similar to Glaeser, Gyourko, and Saiz (2008), incur a linear utility penalty equal to the housing price \( P \) if they choose to live in the neighborhood and thus have to buy a house. Here, we treat all housing units as homogenous with the same price. Given that households have Cobb-Douglas preferences over their consumption, they are effectively risk-neutral at \( t = 1 \), and their utility flow is their expected payoff, or the value of their final consumption bundle less the cost of housing.\(^{1213}\) Each household makes its neighborhood choice subject to a participation constraint that its expected utility from moving into the neighborhood \( E[U_i|I_i] - P \) must (weakly) exceed a reservation utility, which, as in Glaeser, Gyourko, and Saiz (2008), we normalize to 0:

\[
\max \{ E[U_i|I_i] - P, 0 \}.
\]

One can interpret the reservation utility as the expected value of getting a draw of productivity from another potential neighborhood less the cost of search. The choice of neighborhood is made at \( t = 1 \) subject to each household’s information set \( I_i = \{ A_i, P, Q \} \), which includes its private productivity signal \( A_i \), the public signal \( Q \), and the housing price \( P \).\(^{14}\)

### 2.2 Capital Producers

In addition to households, there is a continuum of risk-neutral capital producers that develop capital at \( t = 1 \), and sells this capital to households for their production at \( t = 2 \).

\(^{12}\)For simplicity, our model does not incorporate resale of housing after \( t = 2 \). As a result, we cannot simply deduct the housing price \( P \) as the housing cost from the household’s budget constraint at \( t = 2 \). Instead, we treat the housing cost as a separate utility cost proportional to the housing price at \( t = 1 \). This utility cost captures the notion that a higher housing price implies a greater housing cost without explicitly accounting for different components of the cost, such as initial cost of purchase, cost of mortgage loan, and later resale value.

\(^{13}\)While we focus on a static setting, introducing dynamics would reinforce our amplification mechanism stemming from learning. Since future housing prices are related to aggregate productivity growth in the neighborhood, households most optimistic about moving into the neighborhood because of trading opportunities today would also be the most optimistic in speculating about the value of selling their house to other households in the future.

\(^{14}\)We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.
Similar to many macroeconomic models, such as Bernanke, Gertler, and Gilchrist (1999), we model capital producers as a separate sector in the neighborhood, although we match their population with households to simplify aggregation. This introduces a market-wide supply curve for capital, and consequently a market-wide price, at $t = 2$, while avoiding introducing a speculative retrade motive into households’ capital accumulation decisions.

The representative producer cares about the price of capital at $t = 2$, $R$, which depends on capital’s marginal productivity. This, in turn, depends on the strength of the neighborhood, and which households choose to live in the neighborhood. As a consequence, the housing price in the neighborhood serves as a useful signal to the producer when deciding how much capital to develop at $t = 1$. We assume that each capital producer can develop $K$ units of capital by incurring a convex effort cost $\frac{1}{\lambda} K^\lambda$, where $\lambda > 1$.

While households buy capital from capital producers at $t = 2$, capital producers must forecast this demand when choosing how much capital $K$ to develop at $t = 1$, in order to maximize its expected profit:

$$\Pi_c = \sup_K E \left[ RK - \frac{1}{\lambda} K^\lambda \bigg| I^c \right]$$

where $I^c = \{P, Q\}$ is the public information set, which includes the housing price $P$ and the public signal $Q$. It then follows that the optimal choice of capital sets the marginal cost, $K^{\lambda-1}$, equal to the expected price, $E [R | I^c]$:

$$K = E [R | I^c]^{\frac{1}{\lambda - 1}}.$$

The realized housing price affects the expectation of capital producers about the neighborhood’s strength $A$, which, in turn, impacts their choice of how much capital to develop. As a consequence, in addition to altering the neighborhood choice of potential household entrants, informational frictions in the housing market may also distort investment in the neighborhood.

### 2.3 Home Builders

There is a population of home builders, indexed on a continuum $[0, 1]$, in the neighborhood. Builder $i \in [0, 1]$ builds a single house subject to a disutility from labor

$$e^{-\frac{1}{\pi \tau} \omega_i S_i},$$
where $S_i \in \{0, 1\}$ is the builder’s decision to build and

$$\omega_i = \xi + e_i$$

is the builder’s productivity, which is correlated across builders in the neighborhood through $\xi$. We assume that $\xi = k\zeta$, where $k > 0$ is a constant parameter, and $\zeta$ represents an unobserved, common shock to building costs in the neighborhood. From the perspective of households and builders, $\zeta \sim N(\bar{\zeta}, \tau_\zeta^{-1})$. Then, $\xi = k\zeta$ can be interpreted as a supply shock with normal distribution $\zeta \sim N(\bar{\zeta}, k^2\tau_\zeta^{-1})$ with $\bar{\zeta} = k\bar{\zeta}$. Furthermore, $e_i \sim N(0, \tau_e^{-1})$ such that $\int e_i d\Phi(e_i) = 0$ by the Strong Law of Large Numbers.

At $t = 1$, each builder maximizes his profit

$$\Pi_i(S_i) = \max_{S_i} \left( P - e^{-\frac{1}{1+k}\omega_i} \right) S_i. \quad (5)$$

Since builders are risk-neutral, each builder’s optimal supply curve is

$$S_i = \begin{cases} 
1 & \text{if } P \geq e^{-\frac{k\zeta+e_i}{1+k}}, \\
0 & \text{if } P < e^{-\frac{k\zeta+e_i}{1+k}}. 
\end{cases} \quad (6)$$

The parameter $k$ measures the supply elasticity of the neighborhood, which can arise, for instance, from structural limitations to building or zoning regulation. In the housing market equilibrium, the supply shock $\xi$ not only affects the supply side of the housing market but also demand, as it acts as informational noise in the price signal when households use the price to learn about the common productivity $A$. The elasticity parameter $k$ determines the amount of this informational noise in the price signal.

Although convenient for tractability, and standard in the noisy rational expectations literature, our specification of the housing supply curve, $S(P) = \Phi\left(\sqrt{\tau_e} \left((1 + k) \log P + \xi\right)\right)$, is not essential for our results. We could instead have considered a more realistic model of housing supply with three neighborhoods: one with a perfectly inelastic housing supply, one with a perfectly elastic housing supply, and one in which housing supply is price-elastic and subject to noisy supply shocks. As supply is fixed in the perfectly inelastic neighborhood, housing prices reflect only the housing demand fundamental, and are therefore fully revealing about the strength of the neighborhood $A$. In contrast, since additional houses can be built to absorb new housing demand in perfectly elastic areas, housing prices always equal the marginal cost of building, and contain no information about $A$. It is in intermediate elasticity areas, where prices are driven by both demand and noisy supply-side factors, that households and capital producers face a nontrivial filtering problem in inferring $A$ from housing
prices. Informational frictions are consequently most severe in areas of intermediate supply elasticity, a key feature captured in our more stylized model of housing supply.

2.4 Noisy Rational Expectations Cutoff Equilibrium

Our model features a noisy rational expectations cutoff equilibrium, which requires clearing of the two real estate markets that is consistent with the optimal behavior of households, home builders and capital producers:

- **Household optimization:** each household chooses $H_i$ at $t = 1$ to solve its maximization problem in (3), and then chooses $\{C_j(i)\}_{i \in N}, l_i, K_i$ at $t = 2$ to solve its maximization problem in (2).

- **Capital producer optimization:** the representative producer chooses $K$ at $t = 1$ to solve its maximization problem in (4).

- **Builder optimization:** each builder chooses $S_i$ at $t = 1$ to solve his maximization problem in (5).

- **At $t = 1$, the housing price $P$ clears the housing market:**

\[
\int_{-\infty}^{\infty} H_i(A_i, P, Q) d\Phi(\varepsilon_i) = \int_{-\infty}^{\infty} S_i(\omega_i, P, Q) d\Phi(e_i),
\]

where each household’s housing demand $H_i(A_i, P, Q)$ depends on its productivity $A_i$, the housing price $P$, and the public signal $Q$, and each builder’s housing supply $S_i(\omega_i, P, Q)$ depends on its productivity $\omega_i$, the housing price $P$, and the public signal $Q$. The demand from households and supply from builders are integrated over the idiosyncratic components of their productivities $\{\varepsilon_i\}_{i \in [0,1]}$ and $\{e_i\}_{i \in [0,1]}$, respectively.

- **At $t = 2$, the consumption good price clears the market for each household’s good:**

\[
C_i(i) + \int_{N/i} C_j(j) dj = e^{A_i}K_i^{\alpha \eta_i^{1-\alpha}}, \ \forall \ i \in N,
\]

and the capital price $R$ clears the market for capital:

\[
\int_{N} K_i di = K \int_{N} di,
\]

where $\int_{N} di$ represents the population of households that live in the neighborhood.
3 Equilibrium

In this section, we analyze the housing market equilibrium. We first analyze each household’s optimization problem given in (2), by conjecturing that only households with productivity higher than a cutoff $A^*$ enter the neighborhood. We then derive a unique equilibrium cutoff $A^*$ that satisfies the clearing condition of the housing market. Finally, we verify at the end of the section that the derived cutoff equilibrium is the unique rational expectations equilibrium, in which the choice of each household to live in the neighborhood is monotonic with respect to its own productivity $A_i$.

3.1 Choices of Households and Capital Producers

We first analyze household choices. At $t = 2$, households need to make their production and consumption decisions, after the strength of the neighborhood $A$ is revealed to the public, and home builders and capital producers have also made their choices at $t = 1$. Household $i$ has $e^{A_i K_i^{\alpha \beta}_i}$ units of good $i$ for consumption and trading with other households. The following proposition describes the household’s consumption, labor, and capital choices. Its marginal utility of goods consumption also gives the equilibrium goods price.

**Proposition 1** At $t = 2$, households $i$’s optimal goods consumption is

$$C_i (i) = (1 - \eta_c) (1 - \alpha) e^{A_i K_i^{\alpha \beta}_i}, \quad C_j (i) = \frac{1}{\Phi \left( \sqrt{\tau_\epsilon} (A - A^*) \right) \eta_c (1 - \alpha) e^{A_j K_j^{\alpha \beta}_j}},$$

and the price of its produced good is

$$p_i = e^{\frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c (A - A_i)}} + \frac{1}{2} \psi \left( e^{\frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c (A - A_i)}} \right) \frac{1}{\tau_\epsilon^{1/2}} + \frac{\tau_\epsilon^{1/2}}{\sqrt{\tau_\epsilon} (A - A^*)} \eta_c \frac{\Phi \left( \frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c (A - A_i)} \right)}{} \frac{\Phi \left( \sqrt{\tau_\epsilon} (A - A^*) \right)^{\eta_c}}{}.$$

Its optimal labor and capital choices are

$$\log l_i = \frac{1}{1 - \alpha (1 - \alpha) \psi + (1 + \alpha \psi) \eta_c \psi} \frac{1 + \psi}{\eta_c A} + \frac{1 - \eta_c}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{\alpha}{1 - \alpha} \frac{1}{\log R} \log A_i,$$

$$\log K_i = \frac{1}{1 - \alpha (1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \frac{1 + \psi}{\eta_c A} + \frac{(1 + \psi) (1 - \eta_c)}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} A_i$$

$$- \frac{1}{1 - \alpha} \log R + \frac{1}{1 - \alpha} \frac{1 + \psi}{\eta_c \log} \frac{\Phi \left( \frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c (A - A_i)} \right)}{} \frac{\Phi \left( \sqrt{\tau_\epsilon} (A - A^*) \right)^{\eta_c}}{} + h_0.$$
with constants $l_0$ and $h_0$ given in the Appendix. Furthermore, the expected utility of household $i$ at $t = 1$ is given by

$$E \left[ U \left( \{C_j(i)\}_{j \in \mathcal{N}} ; \mathcal{N} \right) \mid \mathcal{T}_i \right] = (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ p_i e^{A_i} K_i^{1-\alpha} \mid \mathcal{T}_i \right].$$

Proposition 1 shows that each household spends a fraction $1 - \eta_c$ of its wealth (excluding housing wealth) on consuming its own good $C_i(i)$ and a fraction $\eta_c$ on goods produced by its neighbors $\int_{\mathcal{N} / i} C_j(i) \, dj$. When $\eta_c = 1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its output relative to that of the rest of the neighborhood. One household’s good is more valuable when the rest of the neighborhood is more productive, as a result of the complementarity in the household’s utility function. Consequently, this proposition demonstrates that the labor chosen by a household is determined by not only its own productivity $e^{A_i}$, but also the aggregate productivity of other households in the neighborhood.

Proposition 1 also reveals that the optimal choice of labor for each household is log-linear with the strength of the neighborhood $A$, its own productivity $A_i$, and the logarithm of the capital price $\log R$. The final (nonconstant) term reflects selection, in that only households with productivity above $A^*$ enter the neighborhood. Since $A$ is the mean of the distribution of household productivity, it appears in this truncation. This proposition also demonstrates that household $i$’s optimal choice of capital has a similar functional form. The household’s optimal labor choice and demand for capital are both increasing in the strength of the neighborhood $A$, because a higher $A$ represents improved trading opportunities with its neighbors, while they are both decreasing in the price of capital, $\log R$.

We now discuss each household’s decision on whether to live in the neighborhood at $t = 1$ when it still faces uncertainty about $A$. As a result of its Cobb-Douglas utility, the household is effectively risk-neutral over its aggregate consumption, and its optimal choice reflects the difference between its expected utility from living in the neighborhood and the cost $P$ of buying a house in the neighborhood. Then, household $i$’s neighborhood decision is given by

$$H_i = \begin{cases} 1 & \text{if } (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ p_i e^{A_i} K_i^{1-\alpha} \mid \mathcal{T}_i \right] \geq P \\ 0 & \text{if } (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ p_i e^{A_i} K_i^{1-\alpha} \mid \mathcal{T}_i \right] < P \end{cases}.$$  

This decision rule supports our conjecture to search for a cutoff strategy for each household, in which only households with productivity above a critical level $A^*$ enter the neighborhood. This cutoff is eventually solved as a fixed point in the equilibrium.
Given each household’s equilibrium cutoff $A^*$ at $t = 1$ and optimal choices at $t = 2$, we can impose market-clearing in the market for capital to arrive at its price $R$ at $t = 2$. Capital producers forecast this price when choosing how much capital to develop at $t = 1$. These observations are summarized in the following proposition.

**Proposition 2** Given $K$ units of capital developed by capital producers at $t = 1$, the price of capital at $t = 2$ takes the log-linear form

$$
\log R = \frac{1 + \psi}{\psi + \alpha} A - (1 - \alpha) \frac{\psi}{\psi + \alpha} \log K + \frac{1 + \psi}{\psi + \alpha} \eta_c \log \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau^{-1/2} + \frac{A - A^*}{\tau^{1/2}} \right)
$$

$$
+ (1 - \alpha) \frac{\psi}{\psi + \alpha} \log \left( \frac{(1 + \psi)(1 - \eta_c)}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau^{-1/2} + \frac{A - A^*}{\tau^{1/2}} \right) \Phi \left( \sqrt{\tau} (A - A^*) \right) + r_0,
$$

with constant $r_0$ given in the Appendix. The optimal supply of capital by capital producers at $t = 1$ is given by

$$
\log K = \frac{1}{\lambda - \alpha \frac{1 + \psi}{\psi + \alpha}} \log \left[ e^{\frac{1 + \psi}{\psi + \alpha} A} \left( \Phi \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau^{-1/2} + \frac{A - A^*}{\tau^{1/2}} \right) \Phi \left( \sqrt{\tau} (A - A^*) \right) \right)^{\psi \frac{(1 - \alpha)}{\psi + \alpha}} + k_0,
$$

with constant $k_0$ given in the Appendix.

Proposition 2 reveals that the capital price $R$ at $t = 2$ is increasing in the strength of the neighborhood $A$ with the last two (nonconstant) terms reflecting selection by households into the neighborhood, and is decreasing in the supply of capital $K$. It also demonstrates that the optimal supply of capital reflects expectations over not only the strength of the neighborhood $A$, but also the impact of truncation from the neighborhood choice of households on the expected price of capital. The expectation term captures not only the expected productivity from the terms-of-trade (relative prices of household goods) in the first ratio, but also the dispersion in labor productivity in the second ratio.

### 3.2 Perfect-Information Benchmark

With perfect information, all households, home builders, and capital producers observe the strength of the neighborhood $A$ when making their respective decisions. Then, the optimal
choice of capital $K$, given in Proposition 2, simplifies to

$$
\log K = \frac{1 + \psi}{\psi + \alpha} A + \frac{1 + \psi}{\psi + \alpha} \left\{ \eta_c \log \left[ \frac{\Phi \left( \frac{A}{(1-\alpha)\psi + (1+\alpha)\psi} \eta_c \tau^{\epsilon - 1/2} + \frac{A}{\tau^{\epsilon - 1/2}} \right)}{\Phi \left( \sqrt{\tau^{\epsilon}} (A - A^*) \right)} \right] \right\} + k_0, \\
+ \frac{(1 - \alpha) \psi}{\psi + \alpha} \log \left[ \frac{\Phi \left( \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi + (1+\alpha)\psi} \eta_c \tau^{\epsilon - 1/2} + \frac{A}{\tau^{\epsilon - 1/2}} \right)}{\Phi \left( \sqrt{\tau^{\epsilon}} (A - A^*) \right)} \right] \right\}
$$

where $\frac{1 + \psi}{\lambda - \alpha \frac{1 + \psi}{\psi + \alpha}} > 0$, since $\lambda - \alpha \frac{1 + \psi}{\psi + \alpha} > \lambda - 1 > 0$. The supply of capital is log-linear with respect to the strength of the neighborhood $A$, with a correction term for the truncation in the household population that occurs because of household selection into the neighborhood. This truncation term reflects two forces. The first is that the price at which households charge each other for their goods $p_i$ is also affected by this truncation, while second reflects that the smaller population has a higher average marginal product of capital than the full population.

We now characterize the neighborhood choice of households and the housing price. Households will sort into the neighborhood according to a cutoff equilibrium determined by the net benefit of living in the neighborhood, which trades off the opportunity of trading with other households in the neighborhood with the price of housing. Despite the inherent nonlinearity of our framework, the following proposition summarizes a tractable, unique rational expectations cutoff equilibrium that is characterized by the solution to a fixed-point problem over the endogenous cutoff of entry into the neighborhood, $A^*$.

**Proposition 3** In the absence of informational frictions, there exists a unique rational expectations cutoff equilibrium, in which the following hold:

1. Given that other households follow a cutoff strategy, household $i$ also follows a cutoff strategy in its neighborhood choice such that

$$
H_i = \begin{cases} 
1 & \text{if } A_i \geq A^* \\
0 & \text{if } A_i < A^*
\end{cases}
$$

where $A^* (A, \xi)$ solves equation (22) in the Appendix.

2. The cutoff productivity $A^* (A, \xi)$ is monotonically decreasing in $\xi$ and increasing (hump-shaped) in $A$ if $\eta_c < (>) \eta_c^*$, where $\eta_c^*$ is given in (23) in the Appendix.
3. The population that enters the neighborhood is monotonically increasing in both $A$ and $\xi$.

4. The housing price takes the following log-linear form:

$$\log P = \frac{1}{1 + k} \left( \sqrt{\frac{\tau_e}{\tau_e}} (A - A^*) - \xi \right).$$

(9)

5. The housing price $P$, and consequently the utility of the household with the cutoff productivity $A^*$, is increasing and convex in $A$.

Proposition 3 characterizes the unique rational expectations cutoff equilibrium in the economy in the absence of informational frictions, and confirms the optimality of a cutoff strategy for each household’s neighborhood choice when other households adopt a cutoff strategy. Households sort based on their individual productivity into the neighborhood, with the more productive, who expect more gains from living in the neighborhood, entering and participating in production at $t = 2$. This determines the supply of labor at $t = 2$, and, through this channel, the price of capital at $t = 2$.

The optimal cutoff $A^*(A, \xi)$, determined by equation (22), represents the productivity of the marginal household who buys a house, and who consequently is indifferent to entering the neighborhood. This implies that the benefit to the marginal household, the expected utility gain from producing and trading with other households, is equal to the cost, or the housing price. With Cobb-Douglas preferences, this benefit is equal to the expected value of the marginal household’s output from production, which is decreasing in the marginal household’s productivity. The housing price, in contrast, is increasing in the marginal household’s productivity, since it is increasing in the size of the population flowing into the neighborhood. The downward sloping benefit and upward sloping cost gives rise to a unique cutoff productivity, and consequently to a unique rational expectations equilibrium.

The proposition also provides comparative statics of the equilibrium cutoff household $A^*(A, \xi)$. This cutoff is decreasing in $\xi$, since a lower housing price causes more households to enter the neighborhood for a given neighborhood strength $A$, and consequently a higher population enters the neighborhood as $\xi$ increases. The cutoff, in contrast, is increasing in neighborhood strength $A$, since a higher $A$ implies a higher housing price and a higher price of capital, but can be humped-shaped if there is sufficient complementarity because the gains from trade for high realizations of $A$ more than offset the increase in prices. Though
the cutoff productivity either increases or is hump-shaped in $A$, more households ultimately enter the neighborhood because a higher $A$ shifts right (in the sense of first order stochastic dominance) the distribution of households more than it moves the cutoff.

Given a cutoff productivity $A^* (A, \xi)$, the housing price $P$ positively loads on the strength of the neighborhood $A$, since a higher $A$ implies stronger demand for housing, and loads negatively on the supply shock $\xi$, reflecting that a discount is needed to ensure that a positive shift in housing supply is absorbed by a larger household population. As one would expect, the cutoff $A^*$ enters negatively into the price since only households above the cutoff sort into the neighborhood. The higher the cutoff, the fewer the households that enter the neighborhood, and the lower the housing price that is needed to clear the market with the lower housing demand. Despite its log-linear representation, the housing price is actually a generalized linear function of $\sqrt{\frac{\tau_1}{\tau_0}} A - \xi$, since $A^*$ is an implicit function of $A$ and $\log P$.

As a result of endogenous selection into the neighborhood, the productivity of the neighborhood is determined by which households choose to live there. The aggregate productivity of the neighborhood $A_N$ is given by

$$A_N = \log \int_{A^*}^{a} e^{A_j} d\Phi (\varepsilon_j) = A + \frac{1}{2} \frac{\tau_1 - 1}{\tau_0} + \log \Phi \left( \frac{\tau_1^{-1/2} + \frac{A - A^*}{\tau_0^{-1/2}}} {\tau_0} \right).$$

The first two terms are what one would expect without neighborhood choice, while the third term reflects that the productivity of the neighborhood is truncated by selection. Importantly, since $A^* = A^* (A, \xi)$, it follows that $A^*$ depends on the housing price in the neighborhood, introducing feedback from housing prices to real decisions. Similar aggregation results exist for total income $\int_{A_N} e^{A_j} p_i K^{1-\alpha} d\Phi (\varepsilon_j)$ and labor supply $\int_{A_N} l_j d\Phi (\varepsilon_j)$.

### 3.3 Equilibrium with Informational Frictions

Having characterized the perfect-information benchmark equilibrium, we now turn to the equilibrium at $t = 1$ in the presence of informational frictions. With informational frictions, households and capital producers must now forecast the strength of the neighborhood $A$, and the price of capital $R$ at $t = 2$, when choosing whether to live in the neighborhood, and when deciding the amount of capital to develop at $t = 1$. Each household’s type $A_i$ serves as a private signal about the strength of the neighborhood $A$. Since types are positively correlated with this common productivity, higher types also have more optimistic expectations about $A$. This feature ensures that each household will follow a cutoff strategy when deciding whether to live in the neighborhood.
As a result of the cutoff strategy used by households, the equilibrium housing price is a nonlinear function of $A$, which poses a significant challenge to our derivation of the learning of households and producers. It is the case, however, that the equilibrium housing price maintains the same functional form as in (9) for the perfect-information benchmark. As a result, the information content of the publicly observed housing price can be summarized by a sufficient statistic $z(P)$ that is linear in $A$ and the supply shock $\xi$:

$$z(P) = A - \sqrt{\frac{\tau_e}{\tau_e}} \xi. \quad (10)$$

In our analysis, we shall first conjecture this linear sufficient statistic, and then verify that it indeed holds in the equilibrium. This conjectured linear statistic helps to ensure tractability of the equilibrium despite that the equilibrium housing price is highly nonlinear.

By solving for the learning of households and capital producers based on the conjectured sufficient statistic from the housing price, and by clearing the aggregate housing demand from the households with the supply from home builders, we derive the housing market equilibrium. The following proposition summarizes the housing price, each household’s housing demand, and the supply of capital in this equilibrium.

**Proposition 4** There exists a unique noisy rational expectations cutoff equilibrium in the presence of informational frictions, in which the following hold:

1. The housing price takes the log-linear form:

$$\log P = \frac{1}{1 + k} \left( \sqrt{\frac{\tau_e}{\tau_e}} (A - A^*) - \xi \right) = \frac{1}{1 + k} \left( \sqrt{\frac{\tau_e}{\tau_e}} (z - A^*) - \bar{\xi} \right). \quad (11)$$

2. The posterior of household $i$ after observing housing price $P$, the public signal $Q$, and its productivity $A_i$ is Gaussian with the conditional mean $\hat{A}_i$ and variance $\hat{\sigma}^2_A$ given by

$$\hat{A}_i = \hat{\tau}^{-1}_A \left( \tau_A \hat{A} + \tau_Q Q + \frac{\tau_e}{\tau_e} \tau_{\xi z} + \tau_{\xi A_i} \right),$$

$$\hat{\tau}_A = \tau_A + \tau_Q + \frac{\tau_e}{\tau_e} \tau_{\xi A},$$

and the posterior of capital producers, after observing housing price $P$ and the public signal $Q$, is also Gaussian with the conditional mean $\hat{A}^c$ and variance $\hat{\sigma}^2_{A^c}$ given by

$$\hat{A}^c = \hat{\tau}^{-1}_A \left( \tau_A \hat{A} + \tau_Q Q + \frac{\tau_e}{\tau_e} \tau_{\xi z} \right),$$

$$\hat{\tau}^c_A = \tau_A + \tau_Q + \frac{\tau_e}{\tau_e} \tau_{\xi A}.\quad (23)$$
3. Given that other households follow a cutoff strategy, household $i$ also follows a cutoff strategy in its neighborhood choice

$$H_i = \begin{cases} 
1 & \text{if } A_i \geq A^* \\
0 & \text{if } A_i < A^*
\end{cases}$$

where $A^*(z,Q)$ solves equation (25) in the Appendix.

4. The supply of capital takes the form:

$$\log K = \frac{1}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \log F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) + \frac{1+\psi}{\psi+\alpha} A^* + k_0,$$

where $F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right)$ is given in the Appendix, and $\log K$ is increasing in the conditional belief of capital producers $\hat{A}^c$.

5. The cutoff productivity $A^*$ is decreasing, while the population entering the neighborhood and the housing price $P$ are increasing, with respect to the noise in the public signal $\varepsilon Q$. These properties also hold with respect to $z$ under a sufficient, although not necessary, condition that 

$$\frac{1+k}{1+\frac{\tau_\varepsilon}{\tau_A + \tau_Q}} \geq \frac{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}}{\alpha \frac{1+\psi}{\psi+\alpha}} \psi + \alpha + (1 - \alpha) \eta_c \sqrt{\frac{\tau_\varepsilon}{\tau_\varepsilon}}.$$

6. The equilibrium converges to the perfect-information benchmark in Proposition 3 as $\tau_Q \to \infty$.

Proposition 4 confirms that, in the presence of informational frictions, each household will optimally adopt a cutoff strategy when other households adopt a cutoff strategy. Informational frictions make the household’s equilibrium cutoff $A^*(z,Q)$ a function of $z(P) = (1+k) \sqrt{\frac{\tau_\varepsilon}{\tau_\varepsilon}} \log P + A^*$, which is a summary statistic of the publicly observed housing price $P$, and the public signal $Q$, rather than $A$ and $\xi$ as in the perfect-information benchmark. This equilibrium cutoff is a key channel for informational frictions to affect the housing price, as well as each capital producer’s decision to develop capital.

Similar to the perfect-information case, the optimal cutoff $A^*(z,Q)$, determined by equation (25), represents the productivity of the marginal household who buys a house, and who

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15 One may notice that a higher degree of complementarity, $\eta_c$, tightens the sufficient condition, while much of our analysis suggests that it amplifies the role of informational frictions. This is because the condition is not necessary, and is derived by omitting terms for which $\eta_c$ is relevant for amplifying the learning effect.
consequently is indifferent to entering the neighborhood. Once again, this implies that the benefit to the marginal household, the expected utility gain from producing and trading with other households, must equal the cost, or the housing price. Although the cost is the same as in the perfect-information case, with incomplete information the expected utility now reflects the posterior beliefs of the marginal household about the neighborhoods’ unobservable demand fundamental, $A$. The private signal of the marginal household is also its productivity, which is correlated with $A$, and therefore its expectation of $A$ is increasing in its productivity. Consequently, as the marginal household shifts down, a lower $A^*(z,Q)$, and more households enter the neighborhood, the expected utility of the marginal household is again decreasing in its productivity, despite the household’s inference problem. Since the housing price is increasing in the marginal household’s productivity, there is again a unique cutoff productivity, and consequently a unique noisy rational expectations equilibrium. With informational frictions, the cutoff is now a function of the observed public signals, $z$ and $Q$, since the underlying fundamental shocks, $A$ and $\xi$, are not directly observed.

In the presence of informational frictions, the demand-side fundamental $A$ and the supply-side shock $\xi$ are not directly observed by the public and, as a result, do not directly affect the housing price and other equilibrium variables. Instead, their equilibrium effects are bundled together in the housing price $P$ through the specific functional form of $z$. Consequently, we can examine the impact of a shock to either $A$ or $\xi$ by analyzing a shock to $z$. The equilibrium housing price in (11) directly implies that
\[
\frac{\partial \log P}{\partial z} = \frac{1}{1 + k} \sqrt{\frac{\tau_e}{\tau_\xi}} \left(1 - \frac{\partial A^*}{\partial z}\right).
\]
That is, depending on the sign of $\frac{\partial A^*}{\partial z}$, the equilibrium cutoff $A^*$ may amplify or dampen the housing price effect of the fundamental shock $z$. Proposition 4 provides a sufficient (although not necessary) condition for $\frac{\partial A^*}{\partial z} < 0$. In this case, there is an amplification effect. This amplification effect makes housing prices more volatile, as highlighted by Albagli, Hellwig, and Tsyvinski (2015) in their analysis of the cutoff equilibrium in an asset market.\(^{16}\) We shall analyze how different model parameters affect this amplification effect in the next section.

In the perfect-information benchmark, the public signal $Q$ has no impact on either the

\(^{16}\)This interesting feature also differentiates our cutoff equilibrium from other type of nonlinear equilibrium with dispersed information, such as the log-linear equilibrium developed by Sockin and Xiong (2015) to analyze commodity markets. In their equilibrium, prices become less sensitive to their analogue of $z$ in the presence of informational frictions. This occurs because households, on aggregate, underreact to the fundamental shock in their private signals because of noise.
equilibrium cutoff $A^*$ or the housing price because both the demand-side fundamental $A$ and the supply-side shock $\xi$ are publicly observable. In the presence of informational frictions, however, $Q$ affects the equilibrium as it impacts agents’ expectations. The equilibrium housing price in (11) demonstrates that
\[
\frac{\partial \log P}{\partial Q} = -\frac{1}{1+k} \sqrt{\frac{\tau_e}{\tau_e}} \frac{\partial A^*}{\partial Q}.
\]
By affecting the households’ expectations of $A$, and consequently their cutoff productivity to enter the neighborhood, the noise in the public signal $Q$ affects the population that enters the neighborhood and the equilibrium housing price $\log P$: $\frac{\partial A^*}{\partial Q} < 0$ and $\frac{\partial P}{\partial Q} > 0$, as proved in Proposition 4. Furthermore, $Q$ also affects the price of capital, as well as each capital producer’s optimal choice of how much capital to develop.

In the next section, we focus our analysis on two key model parameters: the complementarity in households’ consumption and housing supply elasticity. Households’ consumption complementarity reinforces the effects of informational frictions. Without complementarity, a stronger neighborhood, or a higher $A$, is bad news for households, because a higher $A$ raises not only the housing price, but also the price of capital. With complementarity, however, a stronger neighborhood is also good news for households, because it means that other households in the neighborhood are more productive, and therefore represents a better opportunity for trade. In the presence of informational frictions, complementarity gives each household a stronger incentive to learn about $A$, and thus amplifies the potential distortionary effects from such learning.

Supply elasticity also plays an important and nuanced role in the distortionary effects of learning. It is instructive to consider two polar cases for supply elasticity. When supply is infinitely inelastic (i.e., $k \to 0$), housing prices are only determined by the strength of the neighborhood $A$, and prices are fully revealing to households and capital producers. As a result, there is not any distortion from the learning when supply is infinitely inelastic. In contrast, when supply is infinitely elastic (i.e., $k \to \infty$), prices converge to $\log P = -\zeta$, which is driven only by the supply shock.\footnote{Note from equation (25) that $A^*$ remains finite a.s. as $k \to \infty$, allowing us to take the limit.} In this case, prices contain no information about demand, and therefore no information about the strength of the neighborhood. Consequently, the learning from housing price and the potential distortion of such learning both dissipate as supply elasticity approaches infinity. These two polar cases demonstrate an im-
important insight of our model that the distortion caused by learning from housing prices is humped-shaped with respect to supply elasticity.\footnote{These two polar cases also clarify that this insight is not dependent on the particular form of housing supply specified in Section 2.3. While our specification is standard and consistent with existing models in the noisy rational expectations literature, we expect an alternative specification to deliver qualitatively similar results, as it would not alter these two polar cases.}

We conclude this section by establishing that the cutoff equilibria we have characterized, both with informational frictions and with perfect-information, is the unique rational expectations equilibria in the economy. Regardless of the perceived housing policies of other households in the neighborhood, each household with rational expectations will continue to follow a cutoff strategy, which establishes the uniqueness of the cutoff equilibrium. This is summarized in the following proposition.

**Proposition 5** The unique rational expectations cutoff equilibrium is the unique rational expectations equilibrium in the economy.

Proposition 5 strengthens the predictions of our analysis, as they are now the unique predictions for behavior in the neighborhood. In the next section, we explore the cross-sectional implications of our model for noise-driven housing cycles.

## 4 Model Implications

We now investigate the cross-sectional implications of our model regarding how informational frictions propagate to housing markets and local real investment. We illustrate how several key aspects of the neighborhood and its real estate markets vary across two dimensions: 1) supply elasticity $k$, and 2) the degree of consumption complementarity in household utility $c$.

As highlighted in Section 1, there is a surprising non-monotonic relationship between supply elasticity and the housing cycle experienced by different counties during the recent U.S. housing cycle. The degree of complementarity, meanwhile, is an important characteristic of counties that captures the agglomeration and spillover effects that lead firms from different industries to locate near each other. Conventional wisdom suggests that city diversity not only facilitates economic growth, but also mitigates economic volatility (Glaeser et al. (1992), Deller and Wagner (1998)).

While we have analytical expressions for most equilibrium outcomes, the key equilibrium cutoff $A^*$ needs to be solved numerically from the fixed-point condition in equation (25).
We therefore analyze the equilibrium properties of $A^*$ and other variables through a series of numerical illustrations, by using the following benchmark parameters:

$$
\begin{align*}
\tau_A &= 0.5, \quad \tau_\zeta = 2.0, \quad \tau_\varepsilon = 0.2, \quad \tau_Q = 1.0, \quad \eta_\varepsilon = 0.5, \\
\alpha &= 0.33, \quad \psi = 2.5, \quad k = 0.5, \quad \lambda = 1.1, \quad \bar{A} = 0, \quad \bar{\zeta} = 0.
\end{align*}
$$

In particular, for the share of capital in households’ production, we treat it as being similar to capital, and select the typical estimate of $\alpha = 0.33$. For the Frisch elasticity of labor supply, we choose $\psi = 2.5$, which is within the typical range found in the literature. We set $\tau_\zeta$ to be four-fold larger than $\tau_A$ to ensure that with perfect information, the log housing price variance is monotonically declining in supply elasticity, consistent with conventional wisdom. We set $\lambda = 1.1$ to have capital be in elastic supply, and to avoid having strong convexity in its production function. Finally, we choose for the neighborhood fundamentals $A = \zeta = -0.5$, though the qualitative patterns we highlight hold more generically for a wide range of shock values, and we set the noise in the public signal $Q$ to 0.

Our cross-sectional implications represent testable empirical predictions for the housing market that can be applied to the cross-section of the U.S. housing cycle in the 2000s. Since this was a national cycle, there is a rich cross-section along both dimensions, supply elasticity and industry diversity, to reproduce the empirical analogues of our comparative statics. In particular, any non-monotonicity uncovered empirically would support the role of informational frictions in driving not only the cross-section of house price appreciation, but also local migration, housing construction, and real investment behavior. Such non-monotonicity could potentially help rationalize the hump- and U-shaped patterns we uncovered in both housing cycles and new construction when we sorted along supply elasticity in Section 1.

In addition to analyzing economic outcomes, one could, in principle, also examine the cross-sectional patterns we uncover for home buyer sentiment. The literature has suggested several empirical metrics of home buyer sentiment, such as the housing surveys in Case, Shiller, and Thompson (2012), Google search volume indices from Google Trends, and textual analysis of local media reports, as in Soo (2018). Whether optimism originated from noise from the demand or supply side of the housing market can, in principle, be disentangled by sorting these measures along the dimensions of supply elasticity and industry diversity, similar to the tests we propose using economic outcomes.
Figure 2: The response of the equilibrium cutoff productivity to a noise shock $Q$ (the first row) and a fundamental shock $z$ (the second row) across housing supply elasticity (left) and degree of complementarity (right). The dotted line in each panel is for the perfect-information benchmark, while the solid line is for the case with informational frictions.

### 4.1 Equilibrium Cutoff

Our model features an equilibrium cutoff productivity for the marginal household to enter the neighborhood, which hinges on the households’ learning process about the neighborhood’s strength. This, in turn, determines the population flow into the neighborhood, and the dynamics of both housing markets and real investment. As a consequence, the equilibrium productivity cutoff serves as a channel for informational frictions to impact the local economy. In what follows, we focus on the response of the cutoff in the presence of informational frictions to illustrate how noise in public information can give rise to real effects by facilitating widespread optimism among households.

Figure 2 illustrates how the cutoff responds to a random shock to the noise in the public signal $Q$ and to a fundamental shock to $z$, which is the sufficient statistic that bundles the demand and supply shocks. One can interpret the noise in $Q$ as noise in public information, as in Morris and Shin (2002) and Hellwig (2005), or more broadly as housing market optimism, as in Kaplan, Mitman, and Violante (2017) and Gao, Sockin and Xiong (2019). The first row considers a shock to $Q$, by computing the partial derivative of $A^*$ with respect to $Q$ across different values of supply elasticity $k$ in the left panel and degree of complementarity $\eta_c$ in the
right panel. $Q$ has no impact on the equilibrium in the perfect-information benchmark. In the presence of informational frictions, however, the shock affects households’ expectations about $A$, as they use the public signal to infer the value of $A$. By making households more optimistic about $A$, a positive shock to $Q$ raises each household’s utility, and this lowers the cutoff productivity of the marginal household that enters the neighborhood, as formally shown by Proposition 4. This induces a greater population flow into the neighborhood.

Interestingly, this learning effect is stronger when supply elasticity is greater (the upper-left panel of Figure 2), or when the households’ consumption complementarity is greater (the upper-right panel of Figure 2). The former results from the fact that a greater supply elasticity makes the housing price more dependent on supply-side factors, and therefore less informative about the neighborhood’s strength $A$. Consequently, households place a greater weight on the public signal $Q$ in their learning about $A$, and this amplifies the effect of the noise shock to $Q$. The latter result is driven by the greater role that household learning plays as consumption complementarity increases, as a higher complementarity makes each household more concerned about the neighborhood’s strength.

The second row of Figure 2 considers a fundamental shock to $z$. As discussed earlier, this shock can be a demand-side shock to $A$ or a supply-side shock to $\xi$, which are bundled together in $z$ according to (10) as a result of the presence of informational frictions. Proposition 4 has generally shown that $\frac{\partial A^*}{\partial z}$ can be positive or negative, and provides a sufficient condition for it to be negative. Interestingly, the left panel shows that $\frac{\partial A^*}{\partial z}$ has a U-shape with respect to supply elasticity. It is particularly negative when supply elasticity is in an intermediate value around 0.5, and turns positive when supply elasticity rises roughly above 1.8. This U-shape originates from the aforementioned, non-monotonic learning effect of the housing price. Households use the housing price as a key source of information in their learning about the neighborhood strength $A$, and this learning effect is strongest when supply elasticity has an intermediate value, which makes the equilibrium cutoff particularly sensitive to the $z$ shock. The negative value of the effect implies that the cutoff productivity falls in response to the better neighborhood fundamental, resulting in more households entering the neighborhood despite the higher housing price. The right panel further illustrates that $\frac{\partial A^*}{\partial z}$ decreases monotonically with the degree of complementarity. Specifically, $\frac{\partial A^*}{\partial z}$ is positive when complementarity is low, and becomes more negative as complementarity rises. This

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19 In the perfect-information benchmark, as these shocks are separately observable, the equilibrium cutoff directly reacts to the individual shocks, rather than the bundled $z$. 30
pattern confirms our earlier intuition that the learning effect from housing price strengthens with complementarity.

One could, in principle, directly test the effects of learning on population flows across different regions with properly designed measures of these non-fundamental shocks. Our analysis would suggest that non-fundamental shocks, such as the noise demand shock, have a greater impact in inducing stronger population inflow to areas with greater industry diversity and supply elasticity. Our model also implies that fundamental shocks, stemming either from the demand side or the supply side of the local housing market, have a greater impact on population flow to areas with greater industry diversity and intermediate supply elasticities. The differences in the cross-sectional patterns between the $Q$ and $z$ shocks can also help to distinguish between these two sources of optimism empirically.

4.2 Housing Cycle

This subsection examines the reaction of the housing market to two different shocks, a noise shock to $Q$ and a shock to housing supply $\zeta$. Figure 3 illustrates the impacts of the noise shock to $Q$ on the housing price $P$ and housing stock $S = \int S_i di$, by computing their partial derivatives with respect to $Q$ across different values of supply elasticity $k$ in the two left panels, and across different values of the degree of consumption complementarity $c$ in the two right panels. In the absence of informational frictions, this shock has no effect on the housing market. In the presence of informational frictions, the noise shock raises both the housing price and housing stock (Proposition 4) because it boosts agents’ expectations about the neighborhood’s strength $A$.

Interestingly, the upper-left panel shows that this effect on the housing price is hump-shaped with respect to supply elasticity, and peaks at an intermediate value. This results from the non-monotonicity of the distortionary effect of learning. When housing supply is infinitely inelastic, the noise shock has a muted effect on households’ expectations because the price is fully revealing. When housing supply is infinitely elastic, however, the housing price is fully determined by supply shock and is immune to households’ learning about $A$. As a result, the price distortion caused by household learning is strongest when supply elasticity is in an intermediate range. The lower-left panel further shows that the impact of the noise shock on the housing stock is monotonically increasing with supply elasticity. As discussed earlier in analyzing the equilibrium cutoff $A^*$, the impact of the noise shock to $Q$
Figure 3: The responses of housing price $P$ (top row) and housing stock $S$ (bottom row) to a noise shock to the public signal $Q$ across supply elasticity (left) and degree of complementarity (right). The dotted line in each panel is for the perfect-information benchmark, while the solid line for the case with informational frictions.

on households’ expectations of $A$ is monotonic with respect to supply elasticity, because the informativeness of the housing price is monotonically decreasing in supply elasticity.

The upper-right panel of Figure 3 shows that the effect of the noise shock on the housing price is increasing with respect to consumption complementarity. As complementarity rises, each household cares more about trading goods with other households. This makes households’ expectations about the neighborhood’s productivity a more influential determinant of the housing price. This, in turn, causes the noise shock to have a greater effect on the housing price as complementarity increases. That the growth in housing stock is hump-shaped reflects that near perfect complementarity, almost all households choose to enter the neighborhood and the marginal effect of the increase in the equilibrium cutoff on neighborhood population diminishes.

Next, instead of analyzing a generic shock to $z$, which can be from either the demand side or the supply side, we specifically examine a negative shock to the building cost $\zeta$ (a negative supply shock) to avoid any confusion in interpreting the results. Figure 4 displays the responses of housing price $P$ and housing stock $S$ to this shock across different values of supply elasticity $k$ in the two left panels, and across different degrees of consumption
complementarity $\eta_c$ in the two right panels. In the perfect-information benchmark, the housing price increases with the negative supply shock, and the price increase rises with supply elasticity. In contrast, the housing stock falls with the negative supply shock since the higher housing price discourages more households from entering, and the supply drop is greater when supply elasticity is larger.

In the presence of informational frictions, however, the negative supply shock is, in part, interpreted by households as a positive demand shock when they observe a higher housing price. This learning effect, in turn, pushes up the housing price and the housing stock, relative to the perfect-information benchmark, as shown in the left panels of Figure 4. Across supply elasticity, these distortions are hump-shaped because the impact of learning from the housing price is most pronounced at intermediate supply elasticities, and, consequently, the response of the housing price and housing stock also peak at an intermediate range. As consumption complementarity increases, the learning effect from the negative supply shock is amplified, since households put more weight on the neighborhood’s strength when determining whether to enter the neighborhood. This is shown in the upper-right panel of Figure 4. Similar to the noise demand shock, the impact on the housing stock is hump-shaped, since most households
Figure 5: The responses of capital price $R_1$ at $t = 1$ (top row) and $R$ at $t = 2$ (middle row) and capital stock $K_S$ (bottom row) to a noise shock to the public signal $Q$ across supply elasticity (left) and degree of complementarity (right). The dotted line in each panel is for the perfect-information benchmark, while the solid line for the case with informational frictions.

are already entering the neighborhood as $\eta_c$ nears perfect complementarity.

While our model is static and cannot deliver a boom-and-bust housing cycle across periods, one may intuitively interpret the deviation of the housing price induced by the noise shock and the supply shock from its value in the perfect-information benchmark in Figures 3 and 4 as a price boom, which would eventually reverse. Then, we have testable implications for housing cycles—shocks, such as the noise shock and the supply shock, can lead to more pronounced housing cycles in areas with intermediate housing supply elasticities. This implication is already confirmed by the stylized facts presented in Section 1. Our model also implies that the magnitudes of housing price boom and bust are monotonically increasing with industry diversity, while new housing supply has a hump-shaped relationship with industry diversity. We will further explore this relationship between housing cycle and industry diversity in Section 5.
4.3 Real Investment Cycle

By impacting agents’ expectations, informational frictions not only distort the housing price and housing stock but also other investment decisions related to the neighborhood. The market for capital featured in our model allows us to analyze such effects. We first analyze in Figure 5 how the price and stock of capital react to a noise shock to $Q$ across different values of supply elasticity $k$ in the three left panels, and across the degree of consumption complementarity $\eta_c$ in the three right panels. While households acquire capital only at $t = 2$ at the price $R$, we can also compute the shadow price of capital at $t = 1$, which we denote by $R_1$, as capital producers’ marginal development cost $K^{\lambda-1}$ when they develop capital. This shadow price reflects the producers’ expectations about the price that will prevail at $t = 2$. We examine this shadow price of capital $R_1$ at $t = 1$ in the first row of Figure 5, its market price $R$ at $t = 2$ in the second row, and the stock of capital $K_S = \Phi (\sqrt{\varepsilon} (A - A^*)) K$ built by the producers at $t = 1$ in the third row.

As highlighted earlier, the noise shock $Q$ has no impact on agents’ expectations, and consequently no impact on the price and stock of capital in the perfect-information benchmark. In the presence of informational frictions, however, the noise shock boosts agents’ expectations about $A$. This inflates both the shadow price $R_1$ and the supply of capital $K_S$ at $t = 1$, relative to the perfect-information benchmark. When households buy the capital at $t = 2$, the market price $R$ is determined by their realized productivity, and thus falls to reflect that $A$ had been overestimated at $t = 1$. As a result, the noise shock causes a boom in the market for capital at $t = 1$, in terms of both price and supply, and a bust at $t = 2$ when the price reverses.

The magnitude of this boom-and-bust cycle, as measured by the deviation of the price response at either $t = 1$ or $t = 2$ from the perfect-information benchmark, is monotonically increasing with supply elasticity. As supply elasticity rises, the housing price is driven more by supply side factors, and is therefore less informative about the neighborhood’s strength $A$. Consequently, the public signal $Q$ gets a greater weight in the agents’ learning process about $A$, giving the noise shock to $Q$ a larger impact on the market for capital. With respect to consumption complementarity, the noise shock has a larger impact at lower levels of complementarity. This occurs because a stronger coordination motive lowers the dispersion in households’ production decisions, especially among high productivity households, and
Figure 6: The responses of capital price $R_1$ at $t = 1$ (top row) and $R$ at $t = 2$ (middle row) and capital stock $K_S$ (bottom row) to a negative supply shock across supply elasticity (left) and degree of complementarity (right). The dotted line in each panel is for the perfect-information benchmark, while the solid line for the case with informational frictions.

leads to more entry by lower productivity households into the neighborhood. As both forces reduce the average marginal product of capital, this lowers their supply and their responsiveness to news about $A$.

We now analyze how the market for capital reacts to a negative supply shock $\zeta$ to the housing market in Figure 6, which shows the responses of capital’s shadow price $R_1$ at $t = 1$ in the first row, its market price $R$ at $t = 2$ in the second row, and its supply $K_S$ at $t = 1$ in the third row, across housing supply elasticity in the left panels and across households’ consumption complementarity in the right panels.

In the perfect-information benchmark, the negative supply shock only impacts the housing price, and, through this channel, the cutoff productivity of the households that enter the neighborhood. As is apparent, this direct effect has only a modest impact on the market for capital. In addition, there is only continuation in the price of capital, as there is no

\[ \frac{1}{2} \left( \frac{1}{\eta_c^2 + (1-\alpha)^2} \right) \left( \frac{1}{\eta_c^2 + (1-\alpha)^2} \right)^2 \tau^{-1} \] that captures the dispersion in household demand for capital at $t = 2$. This term is decreasing in the degree of complementarity $\eta_c$, and dampens the impact of misperception about $A$ on the stock of capital by a factor of about 8 in our numerical example as $\eta_c$ varies from 0 to 1.

---

For instance, the optimal supply of capital by suppliers and the shadow price of capital at $t = 1$ are both driven by a constant variance term $\frac{1}{2} \left( \frac{1}{\eta_c^2 + (1-\alpha)^2} \right) \left( \frac{1}{\eta_c^2 + (1-\alpha)^2} \right)^2 \tau^{-1}$ that captures the dispersion in household demand for capital at $t = 2$. This term is decreasing in the degree of complementarity $\eta_c$, and dampens the impact of misperception about $A$ on the stock of capital by a factor of about 8 in our numerical example as $\eta_c$ varies from 0 to 1.
overreaction and reversal. In the presence of informational frictions, however, its impact on the market for capital is substantially larger. This occurs because the negative supply shock is partially interpreted by capital producers as a positive shock to the neighborhood productivity when they learn from the housing price about the neighborhood’s strength $A$. Consequently, it distorts agents’ expectations about $A$ upward, leading to overoptimism about the local economy. This results in both a higher shadow price and a larger supply of capital at $t = 1$, and a greater price reversal at $t = 2$. The magnitudes of these effects are all hump-shaped with respect to housing supply elasticity, as a result of the hump-shaped distortion to agents’ expectations that arises from their learning from the housing price. Similar to the noise demand shock, the negative supply shock distorts the market for capital by leading to overoptimism about $A$, and it is most pronounced at low levels of consumption complementarity.

Our analysis shows that shocks to the housing market can lead to not only a housing cycle, but also to a boom and bust in local real investment. In the context of commercial real estate, this concurrent boom and bust is consistent with Gyourko (2009a) and Levitin and Wachter (2013), which highlight that the recent U.S. housing cycle was accompanied by a similar boom and bust in commercial real estate. It is difficult to simply attribute this commercial real estate boom to the subprime credit expansion that had played an important role for the housing boom, as this credit expansion was mainly targeted at households. In addition, while a run-up in the housing market can inflate commercial real estate prices if there is scarcity in developable land, as in Rosen (1979) and Roback (1982), such a boom would crowd out commercial real estate investment if it is driven by non-fundamental demand. In contrast, both the housing and commercial real estate markets experienced an expansion in construction along with the run-up in prices during the mid-2000s. One may instead attribute this joint cycle to widespread optimism, and our model provides a coherent explanation for the shared optimism in both housing and commercial real estate markets.

5 Additional Evidence

In this section, we provide additional evidence that is consistent with our model implications regarding the relation between housing cycles and industry diversity during the recent U.S. housing cycle of the 2000s. Our model highlights industry diversity as an important characteristic that shapes the amplification of shocks to local housing markets. In the absence of
Figure 7: The U.S. housing cycle in 2000s across counties with different industry diversity. The solid and cross dots represent counties outside and inside the sand states (Arizona, California, Florida and Nevada), respectively. The solid line is the spline line for all counties, while the dashed is for counties outside the sand states. 95% confidence intervals are displayed for the full sample.

Informational frictions, the dotted lines in the right panels of Figure 3 show that a noise shock has no effect on the housing market, and the right panels of Figure 4 show that a negative supply shock has a U-shaped impact on housing supply across industry diversity during the boom period. Consistent with Jacobs (1969) and Glaeser et al. (1992), this U shape suggests that the role of industry diversity is most pronounced at intermediate levels of consumption complementarity: for low consumption complementarity in our model, households put little weight on aggregate behavior in their decisions, while at high consumption complementarity, they put little weight on their own productivity. As such, the most pronounced impact of a negative housing supply shock will occur in the middle, where the loss of marginal households from the initial shock further reduces the housing demand for the remaining population. For the same reason, there is also a hump-shaped cross-sectional pattern in the housing price...
boom and bust with respect to industry diversity. In sharp contrast to these non-monotonic patterns for the benchmark case with perfect information, Figures 3 and 4 show that, in the presence of informational frictions, there are monotonically increasing patterns in the effects of the noise shock and the negative supply shock on the magnitudes of the housing price boom and bust with respect to industry diversity, and hump-shaped patterns in the effects of these shocks on new housing supply during the boom. These more nuanced patterns arise because complementarity, or the benefit from interacting with other households, exacerbates the feedback from learning in the presence of informational frictions.

Figure 7 examines how housing price boom and bust and new housing construction varied across counties with different industry diversity. To measure industry diversity, we follow Glaeser et al. (1992) and construct the city diversity measure as the fraction of the county’s employment in the industries other than its largest five in 2000. The county level employment information across industries is obtained from County Business Pattern (CBP) data in the Census Bureau. The higher is this ratio, the more diverse is the county, which we interpret as having more diversity across industries. We use the same data as our analysis in Section 1.

The three panels in Figure 7 provide scatter plots of the housing price change during the boom period, the housing price change during the bust period, and new housing permits during the boom in different counties against industry diversity, respectively. The patterns match nicely with our model implications—the magnitudes of the price boom and bust appear to be monotonically increasing across industry diversity, while new construction is hump-shaped. To the extent that these patterns cannot occur in the benchmark case with perfect information, Figure 7 confirms the empirical relevance of industry diversity for the impact of learning on housing markets.

6 Conclusion

We introduce a model of information aggregation in housing markets, and examine its consequences for not only housing prices, but also local economic outcomes such as new housing construction and real investment in capital. Our framework offers rich empirical predictions for the neighborhood’s response to shocks originating from both demand and supply side factors in the presence of informational frictions across supply elasticity and the degree of industry diversity. Such predictions can help rationalize the puzzling non-monotonic patterns
that we uncover empirically across counties in the recent U.S. housing cycle.

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**Appendix  Proofs of Propositions**

A.1 Proof of Proposition 1

The first order conditions of household $i$’s optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are

$$C_i(i) : \frac{1 - \eta_c}{C_i(i)} U \left( \{C_k(i)\}_{k \in \mathcal{N}} ; \mathcal{N} \right) = \theta_i p_i,$$

(12)

$$C_j(i) : \frac{\eta_c}{\int_{\mathcal{N}/i} C_j dj} U \left( \{C_k(i)\}_{k \in \mathcal{N}} ; \mathcal{N} \right) = \theta_i p_j,$$

(13)
where \( \theta_i \) is the Lagrange multiplier for the budget constraint. Rewriting (13) as

\[
\frac{\eta_c C_j}{\int_{N/i} C_j dj} U \left( \{ C_k (i) \}_{k \in \mathcal{N}} ; \mathcal{N} \right) = \theta_i p_j C_j
\]

and integrating over \( \mathcal{N} \), we arrive at

\[
\eta_c U \left( \{ C_k (i) \}_{k \in \mathcal{N}} ; \mathcal{N} \right) = \theta_i \int_{N/i} p_j C_j dj.
\]

Dividing equations (12) by this expression leads to \( \frac{\eta_c}{1-\eta_c} = \frac{\int_{N/i} p_j C_j(i) dj}{p_i C_i(i)} \), which in a symmetric equilibrium implies \( p_j C_j (i) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon (A - A^*)})} \frac{\eta_c}{1-\eta_c} p_i C_i (i) \). By substituting this equation back to the household’s budget constraint in (2), we obtain

\[
C_i (i) = (1 - \eta_c) (1 - \alpha) e^{A_i} K_i^{\alpha} l_i^{1 - \alpha}.
\]

The market-clearing for the household’s good requires that

\[
C_i (i) + \int_{N/i} C_i (j) dj = (1 - \alpha) e^{A_i} K_i^{\alpha} l_i^{1 - \alpha},
\]

which implies that \( C_i (j) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon (A - A^*)})} \eta_c (1 - \alpha) e^{A_i} K_i^{\alpha} l_i^{1 - \alpha} \).

The first order condition in equation (12) also gives the price of the good produced by household \( i \). Since the household’s budget constraint in (2) is entirely in nominal terms, the price system is only identified up to \( \theta_i \), the Lagrange multiplier. We therefore normalize \( \theta_i \) to 1. It follows that

\[
p_i = \frac{1 - \eta_c}{C_i (i)} U \left( \{ C_j (i) \}_{j \notin \mathcal{N}} ; \mathcal{N} \right) = \left( e^{A_i} K_i^{\alpha} l_i^{1 - \alpha} \right)^{-\eta_c} \left( \frac{1}{\Phi(\sqrt{\tau_\varepsilon (A - A^*)})} \int_{N/i} e^{A_j} K_j^{\alpha} l_j^{1 - \alpha} dj \right)^{\eta_c}
\]

(14)

Furthermore, given equation (1), it follows since \( C_i (i) = (1 - \eta_c) (1 - \alpha) e^{A_i} K_i^{\alpha} l_i^{1 - \alpha} \) and \( C_j (i) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon (A - A^*)})} \eta_c (1 - \alpha) e^{A_j} K_j^{\alpha} l_j^{1 - \alpha} \) that

\[
U \left( \{ C_k (i) \}_{k \notin \mathcal{N}} ; \mathcal{N} \right) = (1 - \alpha) \left( e^{A_i} K_i^{\alpha} l_i^{1 - \alpha} \right)^{1-\eta_c} \left( \frac{1}{\Phi(\sqrt{\tau_\varepsilon (A - A^*)})} \int_{N/i} e^{A_j} K_j^{\alpha} l_j^{1 - \alpha} dj \right)^{\eta_c}
\]

\[
= (1 - \alpha) p_i e^{A_i} K_i^{\alpha} l_i^{1 - \alpha},
\]

from substituting with the household’s budget constraint at \( t = 2 \).

The first-order conditions for household \( i \)’s choice of \( l_i \) at an interior point is

\[
l_i^\psi = (1 - \alpha) \theta_i p_i e^{A_i} \left( \frac{K_i}{l_i} \right)^{\alpha}.
\]

(15)
from equation (12). Substituting $\theta_i = 1$ and $p_i$ with equation (14), it follows that

$$\log l_i = \frac{1}{\psi + \alpha + (1 - \alpha) \eta_c} \log (1 - \alpha) + \frac{1}{\psi + \alpha + (1 - \alpha) \eta_c} \log \left( (e^{A_i} K_i^\alpha) \left( \int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj \right) \Phi \left( \frac{\sqrt{\tau}}{\epsilon} (A - A*) \right) \right)^{\eta_c}. \tag{16}$$

The optimal labor choice of household $i$, consequently, represents a fixed point problem over the optimal labor strategies of other households in the neighborhood.

Noting that $K_i \equiv \left( \frac{\alpha \psi e^{-A_i l_i^{1-\alpha}}}{R} \right)^{1-\alpha}$ from the first-order condition for $K_i$, we can substitute in the price function $p_i$ to arrive at

$$\log K_i = \frac{1}{1 - (1 - \eta_c) \alpha} \log \left( (e^{A_i l_i^{1-\alpha}})^{1-\eta_c} \left( \frac{1}{\Phi \left( \frac{\sqrt{\tau}}{\epsilon} (A - A*) \right)} \int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj \right)^{\eta_c} \right) \tag{17}$$

which is a functional fixed-point problem for the optimal choice of capital. With some manipulation, by adding a multiple $\frac{1 - (1 - \eta_c) \alpha}{\psi + \alpha + (1 - \alpha) \eta_c}$ of equation (17) to equation (16), we have

$$\log K_i = (1 + \psi) \log l_i - \log \alpha (1 - \alpha) - \log R,$$

and substituting this back into equation (16), we arrive at the functional fixed-point equation

$$A_i + (1 + \alpha \psi) \log l_i = \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} A_i - \frac{(1 + \alpha \psi) \alpha}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} (\log \alpha (1 - \alpha) + \log R) + \frac{1 + \alpha \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} (\log (1 - \alpha) - \eta_c \log \Phi (\frac{\sqrt{\tau}}{\epsilon} (A - A*))) + \frac{(1 + \alpha \psi) \eta_c}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \log \left( \int_{\mathcal{N}/i} e^{A_j} l_j^{1+\alpha \psi} dj \right). \tag{18}$$

Given that household $i$’s optimal labor supply $l_i$ satisfies the functional fixed-point equation (18), let us conjecture for $i$ for which $A_i \geq A^*$, so that $i \in \mathcal{N}$ is in the neighborhood, that

$$\log l_i = l_0 + l_A A + l_s A_i + l_R \log R + l_\Phi \log \frac{\Phi \left( \frac{1 + (\alpha h_s + (1 - \alpha) l_s)}{\tau \tau^{-1/2} + \frac{A - A^*}{\tau \tau^{-1/2}}} \right)}{\Phi \left( \frac{\sqrt{\tau}}{\epsilon} (A - A*) \right)},$$

where $R$ is the rental rate of capital. Substituting these conjectures into the fixed-point recursion for labor, equation (16), we arrive, by the method of undetermined coefficients, at
the coefficient restrictions:

\[
\begin{align*}
l_0 &= \frac{1}{2} \frac{1}{1 - \alpha} \frac{\eta_c}{\psi} \left( \frac{1}{1 - \alpha} \psi + (1 + \alpha \psi) \eta_c \right)^2 \tau_{\varepsilon}^{-1} + \frac{\alpha}{1 - \alpha} \frac{1}{\psi} \log \alpha + \frac{1}{\psi} \log (1 - \alpha), \\
l_A &= \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{\eta_c}{\eta_c} \left( \frac{1}{1 - \alpha} \psi + (1 + \alpha \psi) \eta_c \right), \\
l_s &= \frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c}, \\
l_R &= -\frac{\alpha}{1 - \alpha} \frac{1}{\psi}, \\
l_\Phi &= \frac{1}{1 - \alpha} \frac{1}{\psi},
\end{align*}
\]

which confirms the conjecture. Consequently, we find that, for \( A_i \geq A^* \)

\[
\log l_i = \frac{1}{2} \frac{1}{1 - \alpha} \frac{\eta_c}{\psi} \left( \frac{1}{1 - \alpha} \psi + (1 + \alpha \psi) \eta_c \right)^2 \tau_{\varepsilon}^{-1} + \frac{\alpha}{1 - \alpha} \frac{1}{\psi} \log \alpha + \frac{1}{\psi} \log (1 - \alpha)
\]

\[
+ \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{\eta_c}{\eta_c} A_i + \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{\eta_c}{\eta_c} A_i
\]

\[
- \frac{\alpha}{1 - \alpha} \frac{1}{\psi} \log R + \frac{1}{1 - \alpha} \frac{1}{\psi} \log \frac{\Phi \left( \frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A - A^*}{\tau_{\varepsilon}^{1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)},
\]

and

\[
\log K_i = \frac{1}{2} \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{\eta_c}{\eta_c} \left( \frac{1}{1 - \alpha} \psi + (1 + \alpha \psi) \eta_c \right)^2 \tau_{\varepsilon}^{-1} + \frac{1}{1 - \alpha} \frac{\psi + \alpha}{\psi} \log \alpha + \frac{1}{\psi} \log (1 - \alpha)
\]

\[
+ \frac{1}{1 - \alpha} \frac{(1 + \psi) (1 - \eta_c)}{\eta_c} A_i + \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{1}{\eta_c} A_i
\]

\[
- \frac{1}{1 - \alpha} \frac{\psi + \alpha}{\psi} \log R + \frac{1}{1 - \alpha} \frac{1}{\psi} \frac{\eta_c}{\eta_c} \log \frac{\Phi \left( \frac{1}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A - A^*}{\tau_{\varepsilon}^{1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)}.
\]

Substituting this functional form for the labor supply and capital demand of household \( i \) into equation (14), the price of household \( i 's \) good then reduces to

\[
p_i = e^{\frac{1 + \psi}{1 - \alpha} \psi + (1 + \alpha \psi) \eta_c (A - A_i) + \frac{1}{2} \eta_c (1 + \psi (1 - \alpha) \psi + (1 + \alpha \psi) \eta_c) \tau_{\varepsilon}^{-1} \left( \frac{\Phi \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A - A^*}{\tau_{\varepsilon}^{1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)} \right) \eta_c}.
\]

Finally, that \( U \left( \{C_k (i)\}_{k \in N} ; \mathcal{N} \right) = (1 - \alpha) p_i e^{A_i K_i^{\alpha} l_i^{1 - \alpha}} \), implies

\[
E \left[ U \left( \{C_j (i)\}_{j \in N} ; \mathcal{N} \right) - \frac{l_i^{1 + \psi}}{1 + \psi} \left| I_i \right. \right] = (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ p_i e^{A_i K_i^{\alpha} l_i^{1 - \alpha}} \left| I_i \right. \right].
\]
A.2 Proof of Proposition 2

Substituting the optimal demand for capital $K_i$ into the market-clearing condition for the capital in (7) reveals that the price $R$ is given by

$$
\log R = \frac{1 + \psi}{\psi + \alpha} A - (1 - \alpha) \frac{\psi}{\psi + \alpha} \log K + \frac{1 + \psi}{\psi + \alpha} \eta_c \log \frac{\Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A-A^*}{\tau_{\varepsilon}^{-1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)}
$$

$$
+ (1 - \alpha) \frac{\psi}{\psi + \alpha} \log \frac{\Phi \left( \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A-A^*}{\tau_{\varepsilon}^{-1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)} + r_0,
$$

where $K$ is the total amount of capital developed by capital producers at $t = 1$, and

$$
r_0 = \log \alpha + \frac{1 - \alpha}{\psi + \alpha} \log (1 - \alpha) + \frac{1}{2} \left( \frac{1 + \psi}{\psi + \alpha} \eta_c + (1 - \alpha) \frac{\psi}{\psi + \alpha} (1 - \eta_c)^2 \right) \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \right)^2 \tau_{\varepsilon}^{-1}.
$$

Since market-clearing in the market for capital imposes that $K \int_{i \in \mathcal{N}} dK_i = \int_{i \in \mathcal{N}} K_i dK_i$, it follows from equation (4) that the optimal choice of how much capital that capital producers create is given by equation (8) with constant $k_0$ is given by

$$
k_0 = \frac{\log \alpha + \frac{1 - \alpha}{\psi + \alpha} \log (1 - \alpha) + \frac{1}{2} \left( \frac{1 + \psi}{\psi + \alpha} \eta_c + (1 - \alpha) \frac{\psi}{\psi + \alpha} (1 - \eta_c)^2 \right) \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \right)^2 \tau_{\varepsilon}^{-1}}{\lambda - \alpha \frac{1 + \psi}{\psi + \alpha}}.
$$

A.3 Proof of Proposition 3

We now derive household $i$’s optimal cutoff, given that other households all use an equilibrium cutoff $A^*$. By substituting for prices, the optimal labor and capital choices of household $i$, the realized capital price $R$, and capital demand $K_i$ from Proposition 2, the utility of household $i$ at $t = 1$ from choosing to live in the neighborhood is

$$
E[U_i | I_i] = (1 - \alpha) \frac{\psi}{1 + \psi} e^{u_0 + u_A A + \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} A_i} \left( \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A-A^*}{\tau_{\varepsilon}^{-1/2}} \right) \right)^u \Phi \left( \frac{\Phi \left( \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A-A^*}{\tau_{\varepsilon}^{-1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)} \right)^{u_0} \times \left( \frac{\Phi \left( \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \tau_{\varepsilon}^{-1/2} + \frac{A-A^*}{\tau_{\varepsilon}^{-1/2}} \right)}{\Phi \left( \sqrt{\tau_{\varepsilon}} (A - A^*) \right)} \right)^{(1-\lambda) \frac{1 + \psi}{\lambda - \alpha} \frac{\psi}{\psi + \alpha}}.
$$
where

\[
\begin{align*}
\eta_0 & = \frac{1 + \psi}{2(1 - \alpha)\psi} \left( \lambda \eta_c - \frac{1 - \alpha}{\eta_c} \frac{1 + \psi}{1 + \psi + \alpha} \right) + \frac{1}{\psi} \frac{1 - \alpha}{\eta_c} \frac{1 + \psi}{1 + \psi + \alpha} \left( \frac{1}{\psi} \frac{1 + \psi}{1 + \psi + \alpha} \right), \\
\eta_A & = \frac{1 + \psi}{2(1 - \alpha)\psi} \left( \lambda \eta_c - \frac{1 - \alpha}{\eta_c} \frac{1 + \psi}{1 + \psi + \alpha} \right), \\
\eta_\phi & = \frac{\lambda + \psi}{\lambda - \alpha} \eta_c > 0.
\end{align*}
\]

Since the household with the critical productivity \(A^*\) must be indifferent to its neighborhood choice at the cutoff, it follows that \(U_i - P = 0\), which implies

\[
\begin{align*}
u_i & = \frac{1 + \psi}{2(1 - \alpha)\psi} \left( \lambda \eta_c - \frac{1 - \alpha}{\eta_c} \frac{1 + \psi}{1 + \psi + \alpha} \right) \left( \frac{1}{\psi} \frac{1 + \psi}{1 + \psi + \alpha} \right) \left( \frac{1}{\psi} \frac{1 + \psi}{1 + \psi + \alpha} \right), \\
\nu_A & = \frac{1 + \psi}{2(1 - \alpha)\psi} \left( \lambda \eta_c - \frac{1 - \alpha}{\eta_c} \frac{1 + \psi}{1 + \psi + \alpha} \right), \\
\nu_\phi & = \frac{\lambda + \psi}{\lambda - \alpha} \eta_c > 0.
\end{align*}
\]

which implies the benefit of living with more productive households is offset by the higher cost of living in the neighborhood.

Fixing the critical value \(A^*\) and price \(P\), we see that the LHS of equation (19) is increasing in monotonically in \(A_i\), since \(\frac{1 + \psi}{(1 - \alpha)\psi + (1 + \alpha)\eta_c} (1 - \eta_c) > 0\). This confirms the optimality of the cutoff strategy that households with \(A_i \geq A^*\) enter the neighborhood, and households with \(A_i < A^*\) choose to live somewhere else. Since \(A_i = A + \varepsilon_i\), it then follows that a fraction \(\Phi (\sqrt{\lambda} (A^* - A))\) enter the neighborhood, and a fraction \(\Phi (\sqrt{\lambda} (A^* - A))\) choose to live somewhere else. As one can see, it is the integral over the idiosyncratic productivity shocks of households \(\varepsilon_i\) that determines the fraction of households in the neighborhood.

From the optimal supply of housing by builder \(i\) in the neighborhood (6), there exists a critical value \(\omega^*\):

\[
\omega^* = -(1 + k) \log P,
\]

such that builders with productivity \(\omega_i \geq \omega^*\) build houses. Thus, a fraction \(\Phi (\sqrt{\lambda} (\omega^* - \xi))\) build houses in the neighborhood. Imposing market-clearing, it must be the case that

\[
\Phi (\sqrt{\lambda} (A^* - A)) = \Phi (\sqrt{\lambda} (\omega^* - \xi)).
\]

Since the CDF of the normal distribution is monotonically increasing, we can invert the above market-clearing conditions, and impose equation (20) to arrive at

\[
\log P = \frac{1}{1 + k} \left( \sqrt{\frac{\lambda}{\tau_e}} (A - A^*) - \xi \right).
\]
By substituting for $P$ in equation (19), we obtain an equation to determine the equilibrium cutoff $A^* = A^*(A, \xi)$

$$e^{(1+\psi)(1-\eta_c)+\sqrt{\tau_\epsilon/\tau_e}} A^* \left( \frac{\phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)} + \frac{(1-\lambda)\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha \frac{1+\psi}{\psi+\alpha}} + \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \right) = \frac{1}{\psi(1-\alpha)} e^{\sqrt{\tau_\epsilon/\tau_e} - u_A} \xi - u_0. \tag{22}$$

Taking the derivative of the log of the LHS of equation (22) with respect to $A^*$ gives

$$\frac{d \log LHS}{dA^*} = \frac{1}{\tau_\epsilon^{1/2}} \left( \frac{\phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)} - \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \frac{\phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)} \right) + \frac{1}{1+k} \sqrt{\frac{\tau_\epsilon}{\tau_e}} \frac{(\lambda-1)\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha \frac{1+\psi}{\psi+\alpha}} \left( \frac{\phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_e^{1/2}}\right)} - \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \right).$$

The term in parentheses are nonnegative by the properties of the normal CDF. The last term is nonpositive, since $\lambda > 1$, and attains its minimum at $A^* \to \infty$, from which follows, substituting for $u_\Phi$, that

$$\frac{d \log LHS}{dA^*} \to \frac{1}{1+k} \sqrt{\frac{\tau_\epsilon}{\tau_e}} \frac{\lambda+\psi}{\lambda-\alpha \frac{1+\psi}{\psi+\alpha}} > 0.$$ 

Consequently, since $\frac{d \log LHS}{dA^*} > 0$ when the last term attains its (nonpositive) minimum, it follows that $\frac{d \log LHS}{dA^*} > 0$. Therefore, $\log LHS$, and consequently $LHS$, is monotonically increasing in $A^*$. Since the RHS of equation (22) is independent of $A^*$, it follows that the LHS and RHS of equation (22) intersect at most once. Therefore, the can be, at most, one cutoff equilibrium. Furthermore, since the LHS of equation (22) tends to 0 as $A^* \to -\infty$, and the RHS is nonnegative, it follows that a cutoff equilibrium always exists. Therefore, there exists a unique cutoff equilibrium in this economy.

It is straightforward to apply the Implicit Function Theorem to (22) to obtain

$$\frac{dA^*}{dA} = \frac{1}{1+k} \sqrt{\frac{\tau_\epsilon}{\tau_e}} - \frac{d \log LHS}{dA^*} - u_A \right.$$  

$$\frac{dA^*}{d\xi} = -\frac{1}{1+k} \frac{1}{\frac{d \log LHS}{dA^*} < 0},$$}

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where

\[
\frac{d \log LHS}{dA} = -u_\phi \frac{1}{\tau_\varepsilon - 1/2} \left( \Phi \left( \frac{A-A^*}{\tau_\varepsilon - 1/2} \right) - \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} + A-A^* \right) \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} \right) + \frac{1}{\tau_\varepsilon - 1/2} (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \left( \Phi \left( \frac{A-A^*}{\tau_\varepsilon - 1/2} \right) - \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} + A-A^* \right) \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} \right) \right). 
\]

Note that the nonpositive term in \( \frac{d \log LHS}{dA} \) achieves its minimum at \( A \to -\infty \), at which

\[
\frac{d \log LHS}{dA} \to ((\lambda - 1) \alpha (1-\eta_c) - \lambda \eta_c) \frac{1+\psi}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} (1-\alpha) \psi + (1+\alpha\psi) \eta_c. 
\]

Then, as \( A \to -\infty \), the numerator of \( \frac{dA^*}{dA} \) converges to

\[
\frac{1}{1+k} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} \frac{d \log LHS}{dA} - u_A \to A \to -\infty - \frac{1+\psi}{1+\alpha} \left( \frac{1+\psi}{(1-\alpha) \psi + (1+\alpha\psi) \eta_c} - (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \right),
\]

which is positive. Consequently \( \frac{dA^*}{dA} \bigg|_{A=-\infty} > 0 \). In contrast, as \( A^* \to \infty \), one has that

\[
\frac{1}{1+k} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} \frac{d \log LHS}{dA} - u_A \to A \to -\infty - \frac{1+\psi}{1+\alpha} \left( \frac{1+\psi}{(1-\alpha) \psi + (1+\alpha\psi) \eta_c} - (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \right),
\]

which is negative if

\[
\eta_c > \eta_c^* = (1-\alpha) \frac{\psi}{1+\alpha} \left( \frac{1+\psi}{1+\alpha} \frac{1}{\tau_c} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} + (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \right) + (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \left( \frac{1+\psi}{1+\alpha} \frac{1}{\tau_c} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} \right) - (\lambda - 1) \frac{\alpha_1+\psi}{\psi+\alpha} \left( \frac{1+\psi}{1+\alpha} \frac{1}{\tau_c} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} \right). \tag{23}
\]

We can rewrite equation (22) as

\[
e^{-\frac{1}{(1+\psi)(1-\eta_c)}} (1-\alpha) \psi \frac{1}{1+\psi} \sqrt{\frac{\tau_\varepsilon}{\tau_c}} \left( \Phi \left( \frac{1+\psi}{(1-\alpha) \psi + (1+\alpha\psi) \eta_c} \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} \right) + \Phi \left( \frac{1}{\tau_\varepsilon - 1/2} \right) \right) \left( \Phi \left( \frac{1+\psi}{(1-\alpha) \psi + (1+\alpha\psi) \eta_c} \frac{\tau_\varepsilon - 1/2}{\tau_\varepsilon - 1/2} \right) \right) = \frac{1+\psi}{\psi} e^{1-\alpha} A \frac{1}{\tau_c} \frac{1+\psi}{\psi+\alpha} \]

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where $s = A - A^*$ determines the population that enter the neighborhood. It is straightforward to show that
\[
\frac{d \log LHS}{ds} = - \frac{d \log LHS}{dA^*} < 0.
\]
Consequently, we have
\[
\frac{ds}{d\xi} = - \frac{1}{\frac{1}{1+k} \frac{d \log LHS}{ds}} > 0,
\]
\[
\frac{ds}{dA} = - \frac{\lambda \frac{1}{1-\alpha} \frac{1+\psi}{\psi} \frac{1-\alpha}{\lambda-\alpha} \frac{1+\psi}{\psi+\alpha}}{\frac{d \log LHS}{ds}} > 0.
\]
Thus, the population that enters, $\Phi \left( \sqrt{\tau e} s \right)$, is increasing in $A$ and $\xi$. Furthermore, it follows from (21) that
\[
\frac{d \log P}{dA} = \frac{1}{1+k} \sqrt{\frac{\tau e}{\tau e dA}} > 0,
\]
and therefore the log housing price is increasing in $A$.

Finally, we recognize that
\[
\frac{d^2 P}{dA^2} = \left( \frac{ds}{dA} \right)^2 P + \frac{d^2 s}{dA^2} P = \left( \frac{ds}{dA} \right)^2 P + \frac{\lambda \frac{1}{1-\alpha} \frac{1+\psi}{\psi} \frac{1-\alpha}{\lambda-\alpha} \frac{1+\psi}{\psi+\alpha}}{\left( \frac{d \log LHS}{ds} \right)^2} \frac{ds}{dA} \frac{d^2 \log LHS}{ds^2} P,
\]
where $\lambda \frac{1}{1-\alpha} \frac{1+\psi}{\psi} \frac{1-\alpha}{\lambda-\alpha} \frac{1+\psi}{\psi+\alpha} \frac{ds}{dA} > 0$ by the above arguments. It follows that from calculating $\frac{d^2 \log \text{LHS}}{ds^2}$ that
\[
\lim_{s \to -\infty} \frac{d^2 \log \text{LHS}}{ds^2} = (\lambda (\alpha - \eta_c) - \alpha) \frac{1+\psi}{\psi+\alpha} \frac{1}{\lambda - \alpha} \frac{1}{\sqrt{\tau e}},
\]
and therefore, as $P \to \infty$, from the expression for $\frac{d^2 P}{dA^2}$ one has that $\frac{d^2 P}{dA^2} \to \infty$. Furthermore, as $s \to -\infty$,
\[
\frac{d \log \text{LHS}}{ds} \to - \left( \frac{1}{1+k} \sqrt{\frac{\tau e}{\tau e}} + \frac{\lambda \frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha} \frac{1}{\psi} \right),
\]
and
\[
\lim_{s \to -\infty} \frac{d^2 \log \text{LHS}}{ds^2} = 0,
\]
and $P \to 0$ at an exponential rate. Consequently, as $s \to -\infty$, $\frac{d^2 P}{dA^2} \to 0$. Since $\frac{d^2 P}{dA^2}$ is continuous, it follows that $\frac{d^2 P}{dA^2} \geq 0$. Consequently, $P$ is convex in $A$. Since, in equilibrium, the housing price is equal to the utility of the household with the cutoff productivity, it follows that this utility is also convex and increasing in $A$. 

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A.4 Proof of Proposition 4

Given our assumption about the sufficient statistic in housing price, each household’s posterior about $A$ is Gaussian $A_i | T_i \sim \mathcal{N} \left( \hat{A}_i, \hat{\tau}_i^{-1} \right)$ with conditional mean and variance of

$$
\hat{A}_i = \bar{A} + \tau_A^{-1} \left[ 1 \quad 1 \quad 1 \right] \left[ \begin{array}{ccc} \tau_A^{-1} + \tau_Q^{-1} \\ \tau_A^{-1} \\ \tau_A^{-1} + \tau_Q^{-1} \end{array} \right]^{-1} \left[ \begin{array}{c} Q - \bar{A} \\ \tau_A^{-1} \big( P - \bar{A} \big) \end{array} \right]
$$

$$
\hat{\tau}_i = \tau_A + \tau_Q + z_i^2 \tau_\xi + \tau_\varepsilon.
$$

Note that the conditional estimate of $\hat{A}_i$ of household $i$ is increasing in its own productivity $A_i$. Similarly, the posterior for capital producers about $A$ is Gaussian $A | T^c \sim \mathcal{N} \left( \hat{A}^c, \hat{\tau}^c_i^{-1} \right)$, where

$$
\hat{A}^c = \bar{A} + \tau_A^{-1} \left[ 1 \quad 1 \right] \left[ \begin{array}{cc} \tau_A^{-1} + \tau_Q^{-1} \\ \tau_A^{-1} \end{array} \right]^{-1} \left[ \begin{array}{c} Q - \bar{A} \\ \tau_A^{-1} \big( P - \bar{A} \big) \end{array} \right]
$$

$$
\hat{\tau}^c_i = \tau_A + \tau_Q + z_i^2 \tau_\xi.
$$

This completes our characterization of learning by households and capital producers.

We now turn to the optimal decision of capital producers. Since the posterior for $A - A^*$ of households is conditionally Gaussian, it follows that the expectations in the expression of $K$ in Proposition 2 is a function of the two conditional moments, $\hat{A}^c - A^*$ and $\hat{\tau}^c_i$. Let

$$
F \left( \hat{A}^c - A^*, \hat{\tau}^c_i \right) = E \left[ \frac{e^{(A - A^*)} \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + \frac{A - A^*}{\tau_\varepsilon^{1/2}} \right)^{\eta_c} \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + \frac{A - A^*}{\tau_\varepsilon^{1/2}} \right)^{-\psi(1-\alpha)} \right) ^{1+\psi \psi(1-\alpha)} \right].
$$

Define $z = \frac{A - A^*}{\tau_\varepsilon^{1/2}}$ and the function $f(z)$

$$
f(z) = e^{\tau_\varepsilon^{-1/2} z} \frac{\Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right)^{\eta_c} \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right) \right)^{-\psi(1-\alpha)} \Phi \left( z \right) ^{\psi(1-\alpha)}}{\Phi \left( z \right) ^{\eta_c}},
$$

which is the term inside the bracket in the expectation. Then, it follows that

$$
\frac{1}{f(z)} \frac{df(z)}{dz} = \tau_\varepsilon^{-1/2} + \eta_c \left( \frac{\phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right)^{\eta_c} \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \Phi \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right) \right)^{-\psi(1-\alpha)} \Phi \left( z \right) ^{\psi(1-\alpha)}}{\Phi \left( z \right) ^{\eta_c}} - \frac{\phi \left( z \right)}{\Phi \left( z \right)} \right)
$$

$$
+ \frac{\psi}{1 + \psi} \left( 1 - \alpha \right) \left( \frac{\phi \left( \frac{1+\psi(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right)^{\eta_c} \left( \frac{1+\psi}{(1-\alpha)\psi+(1+\alpha)\psi} \Phi \left( \frac{1+\psi(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha)\psi} \tau_\varepsilon^{-1/2} + z \right) \right)^{-\psi(1-\alpha)} \Phi \left( z \right) ^{\psi(1-\alpha)}}{\Phi \left( z \right) ^{\eta_c}} - \frac{\phi \left( z \right)}{\Phi \left( z \right)} \right).
$$

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Notice that \( \frac{\phi \left( \frac{1+\psi}{1+\alpha} \tau^e - 1/z \right)}{\Phi \left( \frac{1+\psi}{1+\alpha} \tau^e - 1/z \right)} - \frac{\phi(z)}{\Phi(z)} \) achieves its minimum as \( z \to -\infty \). Applying L'Hospital's Rule, it follows that the minimum of \( \frac{1}{f(z)} \frac{df(z)}{dz} \) is given by

\[
\lim_{z \to -\infty} \frac{1}{f(z)} \frac{df(z)}{dz} = -\frac{\tau_e^{-1/2} + \lim_{z \to -\infty} \eta_c}{\phi \left( \frac{1+\psi}{\psi+\alpha+(1+\alpha)\eta_c} \right) - \phi(z)} \\
+ \frac{\psi}{1+\psi} (1-\alpha) \left( \frac{d}{dz} \phi \left( \frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1+\alpha)\eta_c} \tau_e^{-1/2} + z \right) - \frac{d}{dz} \phi(z) \right) \\
= \frac{1+\psi}{\psi+\alpha+(1-\alpha) \eta_c} (1-\eta_c) \tau_e^{-1/2} > 0
\]

from which follows that \( \frac{1}{f(z)} \frac{df(z)}{dz} \geq 0 \) for all \( z \), and therefore \( \frac{df(z)}{dz} \geq 0 \), since \( f(z) \geq 0 \). Consequently, since \( f(z) \frac{1+\psi}{\psi+\alpha} \) is a monotonic transformation of \( f(z) \), it follows that \( \frac{df}{dz}(x, \hat{r}_A) \geq 0 \) since this holds for all realizations of \( A - A^* \). This establishes that the optimal choice of capital is increasing with \( \hat{A}^e \), since \( f(z) \) is increasing for each realization of \( z \).

The optimal choice of \( K \) then takes the following form

\[
\log K = \frac{1}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \log F \left( \hat{A}^e - A^*, \hat{r}_A^e \right) + \frac{1+\psi}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} A^* + k_0
\]

By substituting the expressions for \( K_i \) and \( l_i \) into the utility of household \( i \) given in Proposition 1, we obtain

\[
E \left[ U_i | I_i \right] = (1-\alpha) \psi \frac{e^{\frac{1+\psi}{1+\alpha} (1-\eta_c) \eta_c \psi}}{1+\psi} A_i + \frac{A}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \left( \log F \left( \hat{A}^e - A^*, \hat{r}_A^e \right) + \frac{1+\psi}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} A^* \right) + \frac{1+\psi}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} A^* + u_0
\]

where \( u_0 \) is given in the proof of Proposition 3. When \( A_i = A^* \), this further reduces to

\[
E \left[ U_i | I_i \right] = (1-\alpha) \psi \frac{e^{\frac{1+\psi}{1+\alpha} (1-\eta_c) \eta_c \psi}}{1+\psi} A^* + \frac{A}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \log F \left( \hat{A}^e - A^*, \hat{r}_A^e \right) + u_0
\]
Since the posterior for \( A - A^* \) of household \( i \) is conditionally Gaussian, it follows that the expectations in the expressions above are functions of the first two conditional moments \( \hat{A}_i - A^* \) and \( \hat{r}_A \). Let

\[
G \left( \hat{A}_i - A^* \right, \hat{r}_A \) = E \left[ \left( e^{1 - \frac{1}{\alpha} \psi + \alpha \left( (1 + \psi) \eta_c - \frac{1}{\alpha} \frac{1}{\psi + 1} \right) (A - A^*) \right) \Phi \left( \frac{(1 + \psi) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} \right) \right] \bigg| I_i \right]
\]

Define \( z = \frac{A - A^*}{\tau e} \), and

\[
g(z) = e^{1 - \frac{1}{\alpha} \psi + \alpha \left( (1 + \psi) \eta_c - \frac{1}{\alpha} \frac{1}{\psi + 1} \right) \tau e^{-1/2} z} \Phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} z \right) \Phi(z)^{-\alpha} \]

as the term inside the bracket. Then, it follows that

\[
\frac{1}{g(z)} \frac{dg(z)}{dz} = \frac{1}{1 - \frac{1}{\alpha} \psi} \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{1 + \psi}{\psi + 1} \right) \tau e^{-1/2}
\]

\[
- \eta_c \left( \frac{\phi(z)}{\Phi(z)} - \frac{\phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)}{\Phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)} \right)
\]

\[
+ \alpha \left( \frac{\phi(z)}{\Phi(z)} - \frac{\phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)}{\Phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)} \right)
\]

Note that \( \frac{\phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)}{\Phi \left( \frac{(1 + \psi)(1 - \eta_c) r_{\tau e}^{-1/2}}{(1 - \alpha) \psi + (1 + \alpha) \eta_c} + \frac{A - A^*}{\tau e} + z \right)} = \phi(z) \) achieves its minimum as \( z \to -\infty \). Applying L'Hospital’s Rule, it follows that the minimum of \( \frac{1}{g(z)} \frac{dg(z)}{dz} \) is given by

\[
\lim_{z \to -\infty} \frac{1}{g(z)} \frac{dg(z)}{dz} = \frac{1}{1 - \frac{1}{\alpha} \psi} \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{1 + \psi}{\psi + 1} \right) \tau e^{-1/2}
\]

\[
+ \eta_c \lim_{z \to -\infty} \left( \frac{d}{dz} \phi \left( \frac{1 + \psi}{\psi + 1 - \frac{1}{\alpha} \psi} \right) \left( \frac{1 + \psi}{\psi + 1 - \frac{1}{\alpha} \psi} \frac{1}{\eta_c} \right) - \frac{d}{dz} \phi \left( \frac{1 + \psi}{\psi + 1 - \frac{1}{\alpha} \psi} \right) \right)
\]

\[
+ \alpha \lim_{z \to -\infty} \left( \frac{d}{dz} \phi \left( \frac{1 + \psi}{\psi + 1 - \frac{1}{\alpha} \psi} \right) - \frac{d}{dz} \phi \left( \frac{1 + \psi}{\psi + 1 - \frac{1}{\alpha} \psi} \right) \right)
\]

\[
= \frac{1}{1 - \frac{1}{\alpha} \psi} \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{1 + \psi}{\psi + 1} \right) \tau e^{-1/2}
\]

\[
+ \left( \frac{1 - \eta_c}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \right) \frac{1 + \psi}{\eta_c} \tau e^{-1/2}
\]
With some manipulation, the above expression collapses to
\[ \lim_{z \to -\infty} \frac{1}{g(z)} \frac{dg(z)}{dz} = 0, \]
and it follows that \( \frac{1}{g(z)} \frac{dg(z)}{dz} \geq 0 \), and therefore \( \frac{dg(z)}{dz} \geq 0 \), since \( g(z) \geq 0 \). Consequently, since \( g(z)^{1+\psi} \) is a monotonic transformation of \( g(z) \), it follows that \( \frac{dg}{dx} (x, \hat{\tau}_A) \geq 0 \), since this holds for all realizations of \( A - A^* \).

Since the household with the critical productivity \( A^* \) must be indifferent to its neighborhood choice at the cutoff, it follows that \( U_i - P = 0 \), which implies
\[
\exp \left( \frac{(1+\psi)(1-\alpha \epsilon)}{(1-\alpha)(1+\alpha \epsilon)} \right) A_i + \frac{\alpha}{\lambda - \alpha} \left( \log F(\hat{A}^* - A^*, \hat{\tau}_A) + \frac{1+\psi}{\psi+\alpha} A^* \right) + \frac{1}{\lambda - \alpha} \frac{1+\psi}{\psi+\alpha} \left( \frac{(1+\psi)\eta_c}{(1-\alpha)(1+\alpha \epsilon)\eta_c} \right) \right) A^* + u_0
\]
\[ \cdot G \left( \hat{A}_i - A^*, \hat{\tau}_A \right) = \frac{1}{\psi (1 - \alpha)} P, \quad A_i = A^* \quad (24) \]
which does not depend on the unobserved \( \hat{A} \) or the supply shock \( \epsilon \). As such, \( A^* = A^* (\log P, Q) \).

Furthermore, since \( \hat{A}_i^* \) is increasing in \( A_i \) and \( G(\hat{A}_i^* - A^*, \hat{\tau}_A) \) is (weakly) increasing in \( \hat{A}_i \), it follows that the LHS of equation (24) is (weakly) monotonically increasing in \( A_i \), confirming the cutoff strategy assumed for households is optimal. Those with the RHS being nonnegative enter the neighborhood, and those with it being negative choose to live elsewhere.

It then follows from market-clearing that
\[ \Phi \left( -\sqrt{\tau}(A^* - A) \right) = \Phi \left( -\sqrt{\tau}(\omega^* - \xi) \right). \]

Since the CDF of the normal distribution is monotonically increasing, we can invert the above market-clearing condition, and impose equation (20) to arrive at
\[ \log P = \frac{1}{1+k} \left( \sqrt{\frac{\tau}{\tau_e}} (A - A^*) - \xi \right), \]
from which follows that
\[ z(P) = \sqrt{\frac{\tau}{\tau_e}} ((1 + k) \log P + \xi) + A^* = A - \sqrt{\frac{\tau}{\tau_e}} (\xi - \bar{\xi}), \]
and therefore \( z = \sqrt{\frac{\tau}{\tau_e}} \). This confirms our conjecture for the sufficient statistic in housing price and that learning by households is indeed a linear updating rule.

As a consequence, the conditional estimate of household \( i \) is
\[
\hat{A}_i = \hat{\tau}_A^{-1} \left( \tau_A \hat{A} + \tau_Q Q + \frac{\tau}{\tau_e} \tau \xi z + \tau \xi_A i \right),
\]
\[
\hat{\tau}_A = \tau_A + \tau_Q + \frac{\tau}{\tau_e} \tau \xi + \tau \xi,
\]
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and the conditional estimate of capital producers is
\[ \hat{A}^c = \tau_{A}^{c-1} \left( \tau_{A} \bar{A} + \tau_{Q} Q + \frac{\tau_{e}}{\tau_{e}} \tau_{e} \xi z \right), \]
\[ \hat{\tau}_{A}^c = \tau_{A} + \tau_{Q} + \frac{\tau_{e}}{\tau_{e}} \tau_{e} \xi. \]

Substituting for prices, and simplifying \( A^* \) terms, we can express equation (24) as
\[ e^{\left( \frac{\lambda + \psi}{\lambda - 1} \right) + \frac{\sqrt{\tau_{e}/\tau_{e}}}{1+k}} A^* G \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right) F \left( \hat{A}^c - A^*, \hat{\tau}_{A}^c \right) \left( \frac{1 + \psi}{\psi} \right)^{1 + \frac{\sqrt{\tau_{e}/\tau_{e}}}{1+k}} = 1 + \psi (1 - \alpha) e^{1 + \sqrt{\tau_{e} z - \xi - u_0}}, \]

Notice that the LHS of equation (25) is continuous in \( A^* \). As \( A^* \to -\infty \), the LHS of equation (25) converges to
\[ \lim_{A^* \to -\infty} LHS = 0. \]
Furthermore, by L’Hospital’s Rule and the Sandwich Theorem, one also has that
\[ \lim_{A^* \to \infty} LHS = 1. \]

Since the RHS is independent of \( A^* \), it follows that the LHS and RHS intersect once. Therefore, a cutoff equilibrium in the economy with informational frictions exists.

Now consider the derivative of the log of the LHS of equation (25):
\[ \frac{d \log LHS}{d A^*} = \frac{\lambda + \psi}{\lambda - 1} + \frac{\sqrt{\tau_{e}/\tau_{e}}}{1+k} - \frac{\hat{\tau}_{A}^c}{\tau_{A}} G \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right) F \left( \hat{A}^c - A^*, \hat{\tau}_{A}^c \right) \frac{1 + \psi}{\psi} \frac{1}{\psi} \sqrt{\tau_{e} z - \xi - u_0}, \]
where \( G'(\cdot, \hat{\tau}_{A}) \) and \( F'(\cdot, \hat{\tau}_{A}^c) \) are understood to be first derivatives with respect to the first argument. From our derivation of \( \frac{1}{f(z)} \frac{df(z)}{dz} \) above, we recognize that
\[ \frac{1}{f(z)} \frac{df(z)}{dz} \leq \tau_{e}^{1/2}, \]

since the latter two terms are nonpositive. Furthermore, we can rewrite
\[ \frac{F' \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)}{F \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)} = E \left[ \frac{f \left( \sqrt{\tau_{e}} (A - A^*) \right)}{F' \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)} \left( \frac{d \log f(z)}{dz} \right)_{z = \sqrt{\tau_{e}} (A - A^*)} \right] T^c, \]
where \( E \left[ \frac{f \left( \sqrt{\tau_{e}} (A - A^*) \right)}{F' \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)} \right] T^c = 1 \), so that \( w^f_a \) acts as a weighting function. We can take the derivative inside the expectation because \( f(z) \) has a continuous first derivative. It then follows that
\[ \frac{F' \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)}{F \left( \hat{A}^* - A^*, \hat{\tau}_{A} \right)} \leq \max_A \frac{d \log f(z)}{dz} \left|_{z = \sqrt{\tau_{e}} (A - A^*)} \right| \leq 1. \]
Similarly, rewriting

\[
\frac{1}{g(z)} \frac{dg(z)}{dz} = \frac{1}{1 - \alpha} \psi + \alpha \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{1 + \psi}{\psi + \alpha} \right) \tau^{-1/2}_\varepsilon
\]

and

\[
+ (\alpha - \eta_c) \left( \frac{\phi(z)}{\Phi(z)} - \frac{\phi(1 + \psi(1 - \eta_c)(1 - \alpha)/(1 - \alpha)(\psi + (1 + \alpha \psi) \eta_c) \tau^{-1/2}_\varepsilon + z)}{\Phi(1 + \psi(1 - \eta_c)(1 - \alpha)/(1 - \alpha)(\psi + (1 + \alpha \psi) \eta_c) \tau^{-1/2}_\varepsilon + z)} \right)
\]

it follows, since the last term is nonpositive, that

\[
\frac{1}{g(z)} \frac{dg(z)}{dz} \leq \frac{1}{1 - \alpha} \psi + \alpha \left( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} - \frac{1 + \psi}{\psi + \alpha} \right) \tau^{-1/2}_\varepsilon,
\]

if \( \eta_c \geq \alpha \), since the latter two terms are always nonpositive, and

\[
\frac{1}{g(z)} \frac{dg(z)}{dz} \leq \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \alpha \eta_c \tau^{-1/2}_\varepsilon,
\]

if \( \eta_c < \alpha \), since the second term then attains its maximum as \( z \to -\infty \), and we have truncated the third term. Consequently,

\[
G' \left( \hat{A} - A^*, \frac{\tau}{\hat{A}} \right) \leq \left\{ \begin{array}{ll}
\frac{1}{1 - \alpha} \psi + \alpha & \text{if } \eta_c \geq \alpha \\
\frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \alpha \eta_c & \text{if } \eta_c < \alpha
\end{array} \right.
\]

If \( \eta_c < \alpha \), then, since \( \frac{\tau}{\hat{A}} \leq 1 \) and \( \frac{1 + \psi}{\psi + \alpha} > 1 \), so that \( \frac{\lambda - \alpha}{\lambda - \alpha + \psi} \frac{1 + \psi}{\psi + \alpha} > 1 \), we can bound \( \frac{d \log LHS}{d A^*} \) from below by

\[
\frac{d \log LHS}{d A^*} \geq 1 + \sqrt{\frac{\tau}{\tau_e}} - \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \alpha \eta_c > 0,
\]

since \( \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \alpha \eta_c < 1 \). If \( \eta_c \geq \alpha \), then, since \( \frac{\tau}{\hat{A}} \leq 1 \) and \( \frac{1 + \psi}{\psi + \alpha} > 1 \), so that \( \frac{\lambda - \alpha}{\lambda - \alpha + \psi} \frac{1 + \psi}{\psi + \alpha} > 1 \), we can bound \( \frac{d \log LHS}{d A^*} \) from below by

\[
\frac{d \log LHS}{d A^*} \geq 1 + \frac{\alpha}{1 - \alpha} \frac{1 + \psi}{\psi + \alpha} + \frac{\sqrt{\tau_e/\tau}}{1 + k} - \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \frac{1 + \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \eta_c = \frac{(\psi + \alpha)(1 - \eta_c)}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} + \sqrt{\frac{\tau_e/\tau}{1 + k}} > 0.
\]

Consequently, \( \frac{d \log LHS}{d A^*} \geq 0 \), and therefore the LHS of equation (25) is (weakly) monotonically increasing in \( A^* \). Since the LHS of equation (25) is monotonically increasing in \( A^* \), while
the RHS is fixed, it follows that the cutoff equilibrium is unique. Therefore, there exists a unique cutoff equilibrium with informational frictions.

Since \( \hat{A}^c \) and \( \hat{A}^t \) are both increasing in the public signal \( Q \), it follows by applying the Implicit Function Theorem to equation (25) that

\[
\frac{dA^*}{d\varepsilon_Q} < 0,
\]

where \( \varepsilon_Q \) is the noise in \( Q \), since the LHS of equation (25) is nonnegative and (weakly) monotonically increasing in \( A^* \). Since the noise in the public signal is independent of \( A \), it follows that \( \frac{d\varepsilon}{d\varepsilon_Q} > 0 \), and more households enter the neighborhood in response to a more positive noise shock. Similarly, it also follows that \( \frac{dP}{d\varepsilon_Q} > 0 \), and the housing price increases in response to the stronger housing demand.

By applying the Implicit Function Theorem to equation (25) with respect to \( z \), we see that

\[
\frac{dA^*}{dz} = \frac{1}{1+k} \sqrt{\frac{\tau_e}{\tau_e}} - \frac{G'(\hat{A}^* - A^*, \hat{A}_A)}{G(\hat{A}^* - A^*, \hat{A}_A)} \frac{\tau_A + \tau_Q + \tau_e}{\tau_A + \tau_Q + \tau_e} - \frac{\alpha \frac{1+\psi}{\psi+\alpha} F'(\hat{A}^c - A^*, \hat{A}_A)}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha} F(\hat{A}^c - A^*, \hat{A}_A)} \frac{\tau_A + \tau_Q + \tau_e}{\tau_A + \tau_Q + \tau_e}.
\]

Since \( \frac{G'(\hat{A}^* - A^*, \hat{A}_A)}{G(\hat{A}^* - A^*, \hat{A}_A)} > 0 \), we can find a sufficient condition for the learning effect to dominate the cost effect by truncating the \( \frac{G'(\hat{A}^* - A^*, \hat{A}_A)}{G(\hat{A}^* - A^*, \hat{A}_A)} \) term and recognizing

\[
\frac{F'(\hat{A}^c - A^*, \hat{A}_A)}{F(\hat{A}^c - A^*, \hat{A}_A)} \geq \frac{\alpha \frac{1+\psi}{\psi+\alpha} \psi + \alpha + (1 - \alpha) \eta_c}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} (1 - \eta_c) \frac{\tau_A + \tau_Q + \tau_e}{\tau_A + \tau_Q + \tau_e} \leq 0,
\]

from which follows that it is sufficient, although not necessary, that

\[
\frac{1+\frac{\tau_e}{\tau_A + \tau_Q}}{1+\frac{\tau_e}{\tau_A + \tau_Q} (\tau_A + \tau_Q)} \geq \frac{\lambda - \alpha \frac{1+\psi}{\psi+\alpha} \psi + \alpha + (1 - \alpha) \eta_c}{\alpha (1 - \eta_c)} \frac{\tau_e}{\tau_A + \tau_Q} \sqrt{\frac{\tau_e}{\tau_A + \tau_Q}}.
\]

for \( \frac{dA^*}{dz} < 0 \). As a consequence, more households enter in response to the information in a higher housing price, and this impact is in(de)creasing in \( k \) if \( \frac{\tau_e}{\tau_A + \tau_Q} (\tau_A + \tau_Q) \leq (>) 1 \). It then follows that, in addition, \( \frac{dP}{dz} > 0 \) and more households also enter the neighborhood.

Finally, notice that, as \( \tau_Q \to \infty \), that \( \hat{A}^c \) and \( \hat{A}^t \) converge to \( A \) a.s., since \( \hat{A}_A^c, \hat{A}_A^t \not\to \infty \). Taking the limit along a sequence of \( \tau_Q \), it is straightforward to verify that equation (24) converges to equation (22), and therefore \( A^* \) converges to its perfect-information benchmark value. Taking similar limits for the expressions for capital and labor supply verify that they also converge to their perfect-information benchmark values, and therefore the noisy rational expectations cutoff equilibrium converges to the perfect-information benchmark economy as \( \tau_Q \to \infty \).
A.5 Proof of Proposition 5

We begin with our analysis of the equilibrium at $t = 2$, after informational frictions have dissipated after an arbitrary profile of housing policies by households. To see that this is the unique equilibrium in the economy, define the operator $T : \mathcal{B}^\phi(\mathbb{R}) \to \mathcal{B}^\phi(\mathbb{R})$ characterizing the optimal household $i$'s optimal labor choice:

$$T(x(i)) = \frac{(1 + \alpha \psi)(1 - \eta_c)}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} A_i - \frac{(1 + \alpha \psi) \alpha}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} (\log(1 - \alpha) + \log R)$$

$$+ \frac{1 + \alpha \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \log(1 - \alpha) - \frac{1 + \alpha \psi}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \log E[1_{\{H_j = 1\}}]$$

$$+ \frac{(1 + \alpha \psi) \eta_c}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \log E\left[e^{A_j + x(j)} 1_{\{H_j = 1\}}\right],$$  

(26)

where $\mathcal{B}^\phi(\mathbb{R})$ is the space of functions $\phi$–bounded in the $\phi$–norm $\|f\|_\phi = \sup_{z} \frac{|f(z)|}{\phi(z)}$ for $\phi(z) > 0$. When households follow a cutoff strategy, then $E\left[1_{\{H_j = 1\}}\right] = \Phi\left(\sqrt{\frac{c}{\sigma^2}} (A - A^*)\right)$ and $E\left[e^{A_j + x(j)} 1_{\{H_j = 1\}}\right] = E\left[e^{A_j + x(j)} 1_{\{A_j \geq A^*\}}\right]$. We introduce the weighted norm since $x(i)$ is potentially unbounded. As one can see, $T(x(i))$ is continuous across $i$, since the expectation operator is bounded and preserves continuity for lognormal $A_j$. Furthermore, $T(\cdot)$ satisfies monotonicity $T(y(i)) \geq T(x(i))$ whenever $y(i) \geq x(i)$ ($\forall i$), and discounting since

$$T(x(i) + \beta) = T(x(i)) + \frac{(1 + \alpha \psi) \eta_c}{(1 - \alpha) \psi + (1 + \alpha \psi) \eta_c} \beta < T(x(i)) + \beta \phi(x^*),$$

for $x^* = \arg \sup \|f\|_\phi$ and a constant $\beta > 0$. Therefore, $T$ is a strict contraction map by the Weighted Contraction Mapping Theorem of Boyd (1990). Since a contraction map has, at most, one fixed point, if an equilibrium with a continuous $x(i)$ exists, it must be the unique equilibrium in the economy, at least within the class of functions bounded in the $\phi$–sup norm. Notice now that the choice of $\phi(\cdot)$ is arbitrary, as it does not impact the contractive properties of $T(\cdot)$.

We therefore conclude that the $x(i)$ that solves the fixed-point equation is unique.

Since $x(i) = (1 + \alpha \psi) l_i$ is unique in the economy, it follows that the function for capital $K_i$ is also unique. As such, total capital demand in the economy is unique, and the market-clearing rental rate $R$ is therefore also unique. Consequently, the equilibrium we derived is the unique equilibrium at $t = 2$ in the economy, given the household decision strategy at $t = 1$, $\{H_i\}_{i \in [0,1]}$.

In addition, we recognize from the functional fixed-point equation (26) that $l_i$ is strictly increasing in $A_i$, since one can take a sequence $l_i^k = T l_{i-1}^k$, for which $l_i^k$ is strictly increasing in $A_i$ along the sequence, and take the limit as $k \to \infty$. Furthermore, $l_i$ conditional on $A_i$ is

\[21\] The choice of $\psi(\cdot)$ is not entirely without loss, as existence depends on the space of $\psi$–bounded functions being a complete metric space.
increasing in $A$ from the functional fixed-point equation (26) by similar arguments, since $l_i$ is strictly increasing in $e^{A_j} = e^{A_+e_j}$ for an arbitrary housing policies.

We now turn our attention to $t = 1$. Consider the problem of household $i$ when all other households follow arbitrary strategy profiles. Solving for the household $i$’s optimal consumption and production decisions at $t = 2$, it follows we can express $K_i$ and $p_i$ as

$$K_i = \frac{1}{\alpha (1 - \alpha) R_{i}^{1+\psi}}$$

$$p_i = \left( e^{A_i K_i i} l_i - \alpha \right)^{-\eta_e} \left( \left( 1 + \int_{N_i} e^{A_j K_j i} \right) e^{A_j i} K_j i \right)^{\eta_e}$$

and, by imposing market-clearing in the market for capital, the price of capital is given by

$$R = \frac{1}{\alpha (1 - \alpha) E [1_{H_i = 1}]} \frac{1}{K} \int_{N} l_{j}^{1+\psi} dj.$$  

Since household $i$ is atomistic, it follows, by substituting for $p_i$, $K_i$, and $R$, that

$$E [U_i | I_i]$$

$$= (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ p_i e^{A_i K_i i} l_i - \alpha \right] I_i$$

$$= (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ \left( e^{A_i l_i^{1+\psi}} \right)^{1-\eta_e} E \left[ 1_{H_i = 1} \right]^{\eta_e} \left( \int_{N_i} e^{A_j l_j^{1+\psi}} dj \right)^{\eta_e} \left( \int_{N} l_j^{1+\psi} dj \right)^{-\alpha} K^\alpha \right] I_i$$

$$= (1 - \alpha) \frac{\psi}{1 + \psi} E \left[ \left( e^{A_i l_i^{1+\psi}} \right)^{1-\eta_e} E \left[ e^{A_j l_j^{1+\psi}} | j \in N \right]^{\eta_e} E \left[ l_j^{1+\psi} | j \in N \right]^{-\alpha} K^\alpha \right] I_i. \quad (27)$$

Now fix $K$ as a parameter, since it is publicly observable to all households. Note that the term $e^{A_i l_i^{1+\psi}}$ is increasing in $A_i$, ignoring indirect effects through inference about $A$, and in $A$ conditional on $A_i$, since $l_i$ is increasing in these arguments. Now suppose that $A$ increases to $(1 + \varepsilon) A$, holding fixed $A_i$, $P$, and $K$, and this increases $l_j$ to $(1 + \delta) l_j$ for all $j$. Then

$$\Delta E [U_i | I_i] = (1 + \varepsilon) (1 + \delta)^{1-\alpha} > 0,$$

and $E [U_i | I_i]$ is also increasing in $A$. As all households share a common posterior about $A$ after observing the housing price, their private beliefs about $A$ and their private type $A_j$ are perfectly positively correlated. Consequently, we can express the expected utility of household $i$ as

$$E [U_i | I_i] = E \left[ (1 - \alpha) \frac{\psi}{1 + \psi} \left( e^{A_i l_i^{1+\psi}} \right)^{1-\eta_e} E \left[ e^{A_j l_j^{1+\psi}} | j \in N \right]^{\eta_e} E \left[ l_j^{1+\psi} | j \in N \right]^{-\alpha} K^\alpha | I_i \right]$$

$$= h (A_i, P, Q) K^\alpha,$$

\footnote{In the background, the utility of households, conditional on $A_i$, is supermodular in $A_i$ and the actions of the other households.}
with $\frac{\partial h}{\partial A_i} > 0$ since the argument in the expectation is increasing in $A_i$ and $A$ realization-by-realization.

It then follows that household $i$ will follow a cutoff strategy, and buy if

$$A_i \geq h^{-1}(P/K^0, P, Q),$$

with the cutoff determined by the participation of other households in the neighborhood. This confirms the optimality of their cutoff strategy in their private type. As this holds for any $P$ and $K$, the result follows for any $P$ and $K$.

In the special case of perfect information, we can express $E[U_i | \mathcal{I}_i] = (1 - \alpha) \frac{\psi}{1 + \psi} f(A_i) g(N)$, with $f(A_i) = \left(e^{A_i l_1^{1+\psi}}\right)^{1 - \eta_c}$ is strictly increasing in $A_i$, while $g(N)$ is independent of $A_i$. It then follows that household $i$ will follow a cutoff strategy, and buy if

$$A_i \geq f^{-1}(P/g(N))$$

with the cutoff determined by the participation of other households in the neighborhood.

Furthermore, we recognize that builders, regardless of their beliefs about demand fundamental, $A$, will follow cutoff strategies when choosing whether to supply a house. By market-clearing and rational expectations, the functional form for the housing price and the equilibrium beliefs of households follow.

Given that the housing price has the conjectured functional form, capital producers form rational expectations about $A$, and their optimal supply of capital is unique from the concavity of their optimization program. As such, the cutoff equilibrium we characterized is the unique noisy rational expectations equilibrium in the economy, both with informational frictions and perfect-information.