China's Model of Managing the Financial System*

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May 2020

Abstract

China's economic model involves active government intervention in financial markets. We develop a theoretical framework in which interventions prevent a market breakdown and a volatility explosion caused by the reluctance of short-term investors to trade against noise traders. In the presence of information frictions, the government can alter market dynamics since the noise in its intervention program becomes an additional factor driving asset prices. More importantly, this may divert investor attention away from fundamentals and totally toward government interventions (as a result of complementarity in investors' information acquisition). A trade-off arises: government's objective to reduce asset price volatility may worsen, rather than improve, information efficiency of asset prices.

*We are grateful to Will Cong, Darrell Duffie, Anton Korinek, Debbie Lucas, Helene Rey, Alp Simsek, Larry Summers, Liyan Yang, Hao Zhou, Haoxiang Zhu, and participants of numerous seminars and workshops for helpful comments and suggestions.

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1 Introduction

China's model of "state capitalism" has lifted millions of people out of poverty in the last four decades. It is therefore not surprising that this model has become increasingly appealing to many governments around the globe. While China has adopted many elements of a capitalistic laissez-faire economy that rely on Adam Smith's "invisible hand" since its economic reforms started in the late 1970s, it still relies on heavy-handed interventions by the government.[1] The government's "visible intervening hands" thus interact intensively with the invisible hand and jointly propel the Chinese economy. This paper focuses on the consequences of government interventions in the financial system.

A striking feature of the Chinese financial system is how actively the government leans against short-term market fluctuations in order to promote financial stability. The Chinese government does so through frequent policy changes, using a wide array of policy tools ranging from changes in interest rates and bank reserve requirements to stamp taxes on stock trading, suspensions and quota controls on IPO issuances, modifications to rules on mortgage rates and first payment requirements, and direct trading in asset markets through government-sponsored institutions. For example, during China's stock market turmoil in the summer of 2015, the Chinese government organized a "national team" of securities firms to backstop the market meltdown, as documented by Huang, Miao, and Wang (2019) and Allen et al. (2020). As potential justification for such large-scale, active interventions, China's financial markets are highly speculative[2] and largely populated by inexperienced retail investors. Its markets experience high price volatility and the highest turnover rate among major stock markets in the world.[3]

[1] An intense economic tournament, for instance, motivates local government officials to drive local developments, see e.g., Xu (2011), Qian (2017), and Xiong (2019). Song and Xiong (2018) offer a review of the institutional foundation of China's financial system.

[2] Carpenter and Whitelaw (2017) review an extensive literature on the so-called A-share premium puzzle with the prices of A shares issued by publicly listed Chinese companies to domestic investors trading at substantial price premia and much higher turnover rates, relative to B shares and H shares issued by the same companies to foreign investors. Mei, Scheinkman, and Xiong (2009) attribute this phenomenon to speculative trading of Chinese investors. Furthermore, Xiong and Yu (2011) document a spectacular bubble in Chinese warrants from 2005 to 2008, during which Chinese investors actively traded a set of deep out-of-money put warrants that had zero fundamental value.

[3] In 2008, the China Securities Regulatory Commission issued the China Capital Markets Development Report, which shows that, in 2007, retail accounts with a balance of less than 1 million RMB contributed to 45.9% of stock positions and 73.6% of trading volume on the Shenzhen Stock Exchange. This report particularly highlights speculative behavior of these small investors and the lack of mature institutional investors as important characteristics of China's stock market. Hu, Pan, and Wang (2018) offer a detailed account of stock market volatility and turnover in China.

By leaning against short-term market fluctuations created by inexperienced investors, government intervention helps reduce market volatility and ensure financial stability. Despite the seeming myriad advantages of government intervention, there remain open questions about whether such countercyclical government interventions come with any trade-off and, in particular, whether they are able to ensure the information efficiency of the financial system for investors to invest their savings and for firms to finance their investments. These questions are also relevant for other countries beyond China, as the 2008 global financial crisis has incentivized governments in many countries to intervene in financial markets, though on a smaller scale.

We develop a conceptual framework to analyze these questions. Our analysis focuses on government intervention through direct trading against noise traders in asset markets. We build upon the standard noisy rational expectations models of asset markets with asymmetric information, such as in Grossman and Stiglitz (1980) and Hellwig (1980), and their dynamic versions, including He and Wang (1995) and Allen, Morris, and Shin (2006). In these models, noise traders create short-term price fluctuations, and a group of rational investors, each acquiring a piece of private information, trade against these noise traders to provide liquidity and to speculate on their private information. Our setting includes a new large player, a government, who is prepared to trade against noise traders to stabilize the market.

That the asset fundamental in our setting is unobservable stems from realistic information frictions faced by investors and policy makers in the Chinese economy. Noise traders reflect inexperienced retail investors in the Chinese markets, who contribute to price volatility and instability. The political process, hampered by informational and moral hazard frictions, introduces unintended noise into the government's intervention, with the magnitude of this noise increasing with the intensity of intervention. We assume that investors are myopic so as to reflect the highly speculative nature of Chinese investors. In addition, each investor chooses between acquiring a private signal about either the asset fundamental or this government noise before trading.

With these elements, we build our analysis in several steps. First, we characterize a benchmark economy in which the asset fundamental is observable to the public. In the absence of government intervention, the asset price volatility may explode and the market may break down when the volatility of noise trading becomes sufficiently high. This breakdown occurs because investors care only about the short-term return from trading the asset. The larger the volatility of noise trading, the larger the short-term return volatility and the higher the risk premium that investors demand from trading the asset. This higher risk premium raises the sensitivity of the asset price to noise trading volatility, which further increases short-term return volatility. Through this adverse feedback loop, short-term return volatility eventually explodes when noise trading volatility becomes sufficiently large. The market breaks down because no risk premium exists that can induce investors to trade against noise traders. The volatility explosion and the possible market breakdown introduce a role for the government—which can take a longer horizon than investors—to stabilize the market by trading against noise traders.

We then consider an extended setting in which the asset fundamental is unobservable to both investors and the government. The government follows a linear strategy of trading against perceived noise trading based on the publicly available information, while each investor may acquire a private signal about either the fundamental or the noise in government intervention. Depending on the investors' information choices, we may obtain two different equilibrium outcomes, which we label "fundamental-centric" and "government-centric," respectively. In the fundamental-centric equilibrium, each investor acquires a private signal about the fundamental, and the asset price aggregates their information to partially reveal the fundamental. In contrast, when the government-centric equilibrium arises, investors all focus on learning about noise in government intervention, and their trading, consequently, makes the asset price exposed even to their anticipated future government noise. The likelihood of a government-centric equilibrium increases with the intensity of the government intervention.

Interestingly, for an intermediate range of government intervention intensity, both the fundamental-centric and government-centric equilibria can coexist, as a result of the intertemporal complementarity in investors' information acquisition choices—if investors in the next period acquire fundamental information, the asset price in that next period will be more informative about the asset fundamental, which, in turn, makes it more desirable for investors to acquire information about the asset fundamental. Surprisingly, in the case when both equilibria exist, the same intervention intensity allows the government to achieve substantially lower price volatility in the government-centric equilibrium than in the fundamental-centric equilibrium. This occurs because, in the latter, the government trades both against noise traders, to minimize their price distortion, and against investors, who trade based on their respective private information. In contrast, in the government-centric equilibrium, informed investors share the same information about the fundamental with each other and the government; consequently, they tend to trade alongside the government against noise traders, which reinforces the government's effort to reduce price volatility and renders its intervention more effective in mitigating the price distortion of noise traders. The downside is that the informational efficiency of the asset price is also lower as a result of the lack of information acquisition about the fundamental.

How intensively the government intervenes depends on its objective. There are two similar, albeit subtly different, objectives for government intervention. The first is to reduce asset price volatility and ensure financial stability, while the other is to improve the informational efficiency of the asset price. In a fully microfounded social-welfare analysis, we can trace these two objectives: first, to minimizing the risk premia faced by market participants and second, to improving the efficiency of investment by the real sector of the economy. When investors have no information acquisition choice, the government intervention accomplishes both objectives by simply leaning against noise traders. This "divine coincidence" has often motivated policy makers to treat these two objectives as the same. In practice, the focus is on reducing asset price volatility, as it is easier to measure asset price volatility than informational efficiency; see e.g., Stein and Sundarem (2018). However, once investors' information acquisition choices are endogenous, our analysis shows that the government faces a trade-off between these two seemingly congruent objectives—more intensive interventions can lead to a government-centric equilibrium with lower price volatility, but worse information efficiency.

Our model delivers two key insights not only for government intervention in China, but also more generally for intervention programs in other countries. First, it demonstrates that, even in the absence of informational frictions, there can be a role for government intervention to reduce price volatility and ensure financial stability. Second, and more importantly, our analysis highlights that, despite the seeming appeal for the government to lean against short-term fluctuations in prices, there is a tension between ensuring financial stability and improving informational efficiency. This tension arises because government intervention makes noise in government policy an additional factor in asset prices, which, if sufficiently intensive, may attract the speculation of investors and distract them from acquiring information about fundamentals. This speculation, in turn, reinforces the impact of government noise on asset prices.

The tension between financial stability and informational efficiency captures a key challenge faced by the Chinese government in managing its financial system. As eloquently argued by Zhu (2016), asset bubbles are present in several key sectors of China's financial system, including the real estate market, the bond market, and the stock market, as a result of the paternalistic and countercyclical interventions of the government that create implicit guarantees to investors. Such guarantees embolden investors to ignore economic fundamentals and instead engage in reckless speculation. Our model provides a systematic economic framework to evaluate the consequences of government intervention and, in particular, demonstrates how a policy focus of ensuing financial stability may induce a substantial cost through worsened price efficiency.

Our model builds on the literature that studies information choice in noisy rational expectations models. Hellwig and Veldkamp (2009) demonstrate that, in settings with strategic complementarity in actions, strategic complementarity also arises in information choices, leading agents to choose to learn the same information as others. Ganguli and Yang (2009) and Manzano and Vives (2011) investigate the complementarity in information choice among investors when they can choose to acquire private information either about supply noise or about fundamentals in static settings, and the resulting multiplicity and stability of equilibria. Farboodi and Veldkamp (2016) examine the role of investors' acquisition of information about order flows, instead of fundamentals, in explaining the ongoing trend of increasing price informativeness and declining market liquidity in financial markets. Differing from the intratemporal complementarity in information choices studied by these papers, our model highlights intertemporal complementarity of investors' information choice, in a spirit similar to Froot, Scharfstein, and Stein (1992). They illustrate how intertemporal complementarity in information acquisition can lead investors to focus on learning about short-term, rather than long-term, fundamentals.

Our paper also contributes to the literature on the financial market implications of government intervention. Bond and Goldstein (2015) study the impact on information aggregation in prices when uncertain, future government intervention influences a firm's real outcomes. Cong, Grenadier, and Hu (2017) explore the information externality of government intervention in money market mutual funds in a global games environment in which investors face strategic coordination issues and intervention changes the information publicly available to them. Angeletos, Hellwig, and Pavan (2006) and Goldstein and Huang (2016) consider information design by an informed policy maker that can send messages through its actions to coordinate the response of private agents in a global games setting. In contrast to these studies, we focus on the incentives of market participants to acquire information when there is uncertainty about the scope of government intervention in financial markets through large-scale asset purchases. Our government, by internalizing investors' information acquisition choices, faces a tension between reducing price volatility and improving price efficiency.

The paper is organized as follows. Section 2 provides institutional background. By first taking the government intervention as given, Sections 3 and 4 analyze its effects under perfect information and information frictions, respectively. Section 5 analyzes the government's intervention objective; Section 6 concludes with some additional discussions. We cover the salient features of the model under different settings in the main text, while providing more detailed descriptions of the model in the Appendix. A separate Online Appendix contains all technical proofs involved in our analysis.

2 Institutional Background

As an integral part of the Chinese economy, China's financial system is crucial for channeling capital to real investment. While market forces have played increasingly important roles in driving financial markets, the systemic risk imposed by the financial system on the broader economy has motivated the government to regularly intervene. This section summarizes the extent of government intervention in China's financial system, focusing in particular on the general strategy of the Chinese government to lean against short-term market fluctuations either through direct trading or broad policy interventions.

The national team and the 2015 stock market crash. In 2014-15, the Chinese stock market experienced a dramatic boom-and-bust cycle, as described by Allen et al. (2020). From July 2014 to June 2015, the Chinese stock market index increased by over 150%, which prompted a large number of new investors with little financial knowledge and investment experience to flood into the stock market and motivated, in particular, these investors to take on substantial leverage through margin financing of their stock positions. In June 2015, when the stock market initially plunged by over 30%, many investors received margin calls, which forced them to liquidate their leveraged positions. Bian et al. (2017) provide a systematic account of the resulting margin spiral, which directly threatened the stability of the whole financial system. Motivated by the pressing need to maintain financial stability, the Chinese government organized a "national team" of investment firms to bail out the stock market in the period from June to September of 2015. According to Allen et al. (2020) and Huang, Miao, and Wang (2019), during this bailout period the national team invested in 1,365 stocks, which accounted for about 50% of the total number of listed stocks and 6% of the capitalization of the Chinese stock market. Their analysis shows that, by stablizing the market, the intervention of the national team substantially increased the value of the rescued nonfinancial firms through increased stock demand, reduced default probabilities, and improved market liquidity.

Other policy interventions in the stock market. In addition to the trading of the national team during the 2015 stock market crash, the China Securities Regulatory Commission (CSRC), the regulator of China's stock market, has regularly used a set of policy tools to lean against cycles in the stock market. A common feature of the CSRC's policy interventions is to loosen aggregate stock supply during booms and tighten supply during busts. Since 1994, the CSRC has suspended IPO issuance nine times, usually when the stock market was distressed and sometimes for as long as 15 months. Packer and Spiegel (2016) find a significant, positive relation between the number of IPOs and the market index return in China's stock market, confirming the CSRC's effort to use IPO issuance to lean against the market cycle. During the 2015 stock market turmoil, the CSRC had also employed another measure to stabilize the market: prohibiting large shareholders from selling their shares. As discussed by Allen et al. (2020), on July 8, 2015, the CSRC imposed a lockup on shareholders owning 5% or more of their companies, initially for six months. It was later extended in January 2016 after the stock market declined sharply again.

The Chinese authorities have also used other measures to lean against the stock market cycles. These are not directly related to share supply but may nevertheless have countercyclical effects on the intensity of stock trading. For example, during the 2015 stock market crash, the People's Bank of China (PBC), the central bank, cut interest rates and reduced banks' required reserve ratios to boost the liquidity in the financial system; the CSRC announced a relaxation of margin trading rules, lowering thresholds for individual investors to trade on margins and expanding brokerages' funding channels; and the Shanghai and Shenzhen stock exchanges lowered securities transaction fees by 30%. More generally, the CSRC has changed the rate of transaction tax on stock trading seven times since 1994, increasing the tax rate during market booms and reducing it during market downturns; see e.g., Deng, Liu, and Wei (2018) and Cai et al. (2019).

Countercyclical interventions in other markets. The Chinese government has also actively intervened in other markets besides the stock market. According to Liu and Xiong (2019), the real estate market has perhaps even more systemic importance to the Chinese economy, because of the substantial exposures of local governments, real estate developers, firms, and households, who use real estate assets as collateral for debt financing. As a result, the Chinese government has used a wide range of policy measures to lean against real estate cycles. During booms, the government tends to increase land supply for real estate development. It also restricts purchases of investment homes in large cities, by both residents and nonresidents, and increases mortgage down payments and mortgage rates for purchases of both primary and investment homes. During downturns, the government tends to reverse these measures. Furthermore, the PBC also adopts counter-cyclical monetary policies to assist government efforts to lean against real estate cycles.

The foreign exchange market is another marketplace where the government intervenes. During the past decade, the Chinese government has made great efforts to internationalize its currency RMB and liberalize its capital accounts. This process exposed the RMB exchange rate to intense market speculation and China's capital accounts to dramatic inflows and outflows. In 2013-15, domestic enterprises took on dollar debt from the global capital markets to take advantage of the substantially lower interest rates outside China, leading to large capital inflows. The direction of capital flows reversed after late 2015 when China's economic growth slowed and intense market pressure mounted to speculate against the RMB exchange rate. In the subsequent two years, capital outflows led to China's loss of FX reserves in excess of $1 trillion. In response to these developments, the PBC has adopted a series of macroprudential regulatory measures to lean against speculative capital inflows/outflows. As detailed in the 2018 report of HKEX, during periods of capital outflows or depreciation pressure on the RMB, the PBC adopted the following measures: 1) an increase of the FX risk reserve requirement ratio to 20%; 2) the introduction of reserve requirements on foreign financial institutions' RMB deposits in domestic financial institutions, which directly affects the supply of RMB to foreign speculators for shorting RMB; 3) the use of a countercyclical adjustment factor in the mechanism of determining the RMB's central parity rate; and 4) the imposition of unified regulations on local and foreign currencies. During periods of capital inflows or appreciation pressure on the RMB, the PBC reversed the aforementioned measures.

Government interventions in other countries. Governments of other countries have also intervened in financial markets, albeit not as regularly or broad in scope as the Chinese government. During the Asian financial crisis in the summer of 1998, for instance, the Hong Kong Monetary Authority organized a massive intervention to lean against market speculators by buying large quantities of Hong Kong dollars, individual stocks in the Hong Kong Hang Seng Index (HSI), and HSI futures contracts (Bhanot and Kadapakkam (2006)). In Japan, the Bank of Japan has gradually expanded its stock purchase program in recent years as part of its massive monetary easing program to lift the country out of deflation (Shirai (2018)). More broadly, since the global financial crisis in 2008, several central banks, including the U.S. Federal Reserve and the European Central Bank, have implemented quantitative easing through large-scale asset purchase programs, which are aimed more at stimulating the economy than at leaning against short-term market fluctuations.

3 The Basic Model with Perfect Information

In this section, we present a baseline setting with perfect information to illustrate how government intervention helps avoid market breakdown and the volatility explosion caused by the reluctance of short-term investors to trade against noise traders. Our model can be seen as a generalized version of De Long et al. (1990) with fundamental risk. We will expand the setting in the next section to incorporate realistic information frictions.

Consider an infinite-horizon economy in discrete time with infinitely many periods: $t = 0, 1, 2....$ There is a risky asset, which can be viewed as stock issued by a firm that has a stream of cash flows D_t over time:

$$D_t = V_t + \sigma_D \varepsilon_t^D.$$

The component V_t is a persistent component of the fundamentals, while ε_t^D is independent and identical cash flow noise with a Gaussian distribution of $\mathcal{N}(0, 1)$ and $\sigma_D > 0$ measures the volatility of cash flow noise.

As the literature has extensively studied the direct effects of government policies on the profitability of firms,[4] we intend to analyze a different channel, through which government intervention can impact market dynamics without directly affecting the firm's cash flow. Specifically, we assume that the asset's fundamental V_t follows an exogenous AR(1) process:

$$V_t = \rho_V V_{t-1} + \sigma_V \varepsilon_t^V,$$

where $\rho_V \in (0,1)$ measures the persistence of V_t, $\sigma_V > 0$ measures its volatility, and $\varepsilon_t^V \sim \mathcal{N}(0,1)$ is independently and identically distributed shock.

In this section, we assume that at time t, V_{t+1} is **observable** to all agents in the economy. This setting serves as a benchmark.[5] We will remove this assumption in the next section to make V_{t+1} unobservable to both the government and investors and then discuss how government intervention affects the investors' information acquisition.

For simplicity, suppose there is also a risk-free asset in elastic supply that pays a constant gross interest rate $R^f > 1$. In what follows, we define R_{t+1} to be the excess payoff, not percentage return, to holding the risky asset:

$$R_{t+1} = D_{t+1} + P_{t+1} - R^f P_t.$$

There are three types of agents in the asset market: noise traders, investors, and the government. We describe each of them below.

3.1 Noise Traders

Motivated by the large number of inexperienced retail investors in China's stock markets, we assume that, in each period, these inexperienced investors, whom we call noise traders, submit exogenous market orders into the asset market. This way of modeling noise trading is standard in the market microstructure literature. We denote the quantity of their net *buy* orders by N_t and assume that N_t is an i.i.d. process:

$$N_t = \sigma_N \varepsilon_t^N,$$

[4] For example, if the government faces a time-varying cost in implementing such a policy, the cost of the policy can become an important factor in driving variation in a stock's cash flows and thus its price dynamics. See Pastor and Veronesi (2012, 2013) for recent studies that explore this channel. In addition, when government policies affect the cash flow of publicly traded firms, Bond and Goldstein (2015) show that such intervention feeds back into how market participants trade on their private information. This results in socially inefficient aggregation of private information about the unobservable fundamental v_t into asset prices, which can impede policymaking if the government also infers relevant information about v_t from the traded asset price in determining the scale of its intervention.

[5] We make v_{t+1}, not just v_t, observable at time t so that this benchmark is exactly the limiting case of the setting in the next section, where we allow the precision of each investor's private information about v_{t+1} to become arbitrarily large.

where $\sigma_N > 0$ measures the volatility of noise trading (or noise-trader risk in this market), and $\varepsilon_t^N \sim \mathcal{N}(0,1)$ is independently and identically distributed shocks to noise traders. The presence of noise traders creates incentives for other investors to trade in the asset market.

3.2 Investors' Problem

There is a continuum of investors in the market who trade the asset on each date t. We assume that these investors are myopic. They can be thought of as living for only two periods, trading in the first and consuming in the second. That is, in each period a group of new investors with measure 1 joins the market, replacing the group from the previous period. We index an individual investor by $i \in [0,1]$. Investor i born at date t is endowed with wealth $\bar{W}$ and has constant absolute risk aversion CARA preferences with coefficient of risk aversion γ over its next-period wealth W_{t+1}^i:

$$U_t^i = E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t\right].$$

It purchases X_t^i shares of the asset and invests the rest in the risk-free asset at a constant rate R^f, so that W_{t+1}^i is given by

$$W_{t+1}^i = R^f \bar{W} + X_t^i R_{t+1}.$$

The investors have symmetric, perfect information, and their expectations are all taken with respect to the full-information set $\mathcal{F}_t = \sigma\left(\{V_{s+1}, N_s, D_s\}_{s\leq t}\right)$ in this section. As a result of CARA preferences, an individual investor's trading behavior is insensitive to his initial wealth level.

The assumption of investor myopia follows from De Long et al. (1990) and can be motivated from agency problems faced by institutional investors; see, e.g., Shleifer and Vishny (1997). In our setting, this assumption also serves to capture the short-termism of Chinese investors, which is important for generating market breakdown when noise trader risk becomes sufficiently large.

3.3 Equilibrium without Government Intervention

To facilitate our discussion, we first characterize the rational expectations equilibrium without government intervention. Specifically, we derive the equilibrium price and show formally that market volatility explodes and the market ultimately breaks down when noise trader risk, σ_N, rises above a critical threshold.

We first conjecture a linear rational expectations equilibrium.[6] In this equilibrium, the asset price P_t is a linear function of the fundamental V_{t+1} and the noise trader shock N_t:

$$P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t,$$

where $\frac{1}{R^f - \rho_V} V_{t+1}$ is the expected present value of cash flows from the asset. With this conjected price function, an investor holding the asset faces, at time t, price risk from fluctuations of both V_{t+1} and N_t, as given by

$$Var\left(R_{t+1}|\mathcal{F}_t\right) = \sigma_D^2 + \left(\frac{1}{R^f - \rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2.$$

CARA utility with normally distributed payoffs implies identical asset demand X_t^i:

$$X_t^i = -\frac{1}{\gamma} \frac{p_N R^f}{\sigma_D^2 + \left(\frac{1}{R^f - \rho_V}\right)^2 \sigma_V^2 + p_N^2 \sigma_N^2} N_t,$$

which trades off expected asset return with return variance over the subsequent period.

Then, imposing market-clearing in the asset market $X_t^i = N_t$ leads to a quadratic equation that pins down the price coefficient p_N. There may exist two positive roots for p_N. We focus on the less positive root.[7] The following proposition shows that there may not be any root—i.e., the equilibrium does not exist—if σ_N is higher than a threshold:

$$\sigma_N^* = \frac{R^f}{2\gamma\sqrt{\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2}}. \tag{1}$$

Proposition 1 *If noise-trader risk satisfies $\sigma_N \leq \sigma_N^*$, then an equilibrium exists with $\frac{\partial(Var(R_{t+1}|\mathcal{F}_t))}{\partial \sigma_N^2} > 0$, and $\frac{\partial(Var(R_{t+1}|\mathcal{F}_t))}{\partial \sigma_N^2} \to \infty$ as $\sigma_N \to \sigma_N^*$, which implies that the asset return variance is highest at $\sigma_N = \sigma_N^*$ with a value of $2\left[\sigma_D^2 + \left(\frac{\sigma_V}{R^f - \rho_V}\right)^2\right]$. If $\sigma_N > \sigma_N^*$, no equilibrium exists.*

We provide a proof to Proposition 1 as a special case of Proposition 3. It shows that the asset return variance increases with noise-trader risk, σ_N, and the rate of this increase

[6] We later verify in the proof of Proposition 3 that there cannot be any nonlinear equilibrium if we treat the economy as the infinite-horizon limit of an economy with a finite number of trading periods.

[7] As $\sigma_N \to 0$ (i.e., noise-trader risk vanishes from the economy), the less positive root has the nice property that $p_N \sigma_N \to 0$ (i.e., the price impact of noise traders diminishes), while the more positive root diverges. Furthermore, if one treats the quadratic equation defining p_N as a recursion, the less positive root is backward stable while the more positive root is forward stable, and market breakdown occurs when both roots diverge ($p_N \to \infty$). The less positive root is consequently the more intuitive root since prices are determined by backward, rather than forward, induction of future payoffs.

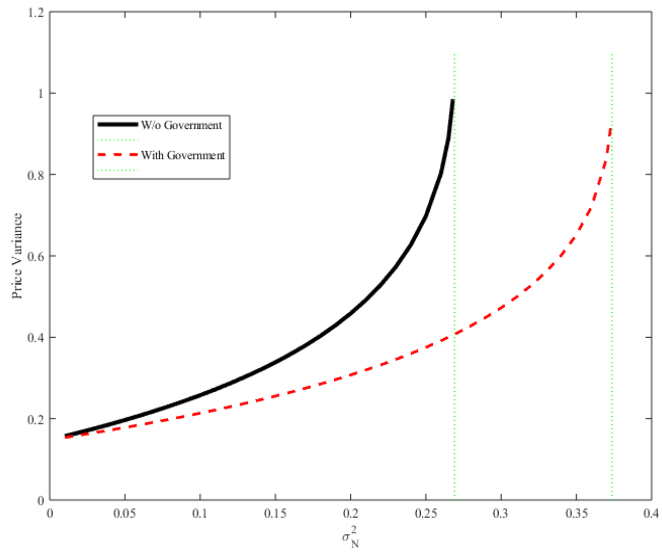

Figure 1: Asset price variance with and without government intervention with respect to the variance of noise trading σ_N^2. The solid line represents the case without government intervention, and the dashed line represents the case with government intervention at a given intensity ϑ_N, based on the following parameters: $\gamma = 1$, $R^f = 1.01$, $\rho_v = 0.75$, $\sigma_v^2 = 0.01$, $\sigma_D^2 = 0.8$, $\vartheta_N = 0.2$.

explodes as σ_N gets close to the critical threshold σ_N^*, as illustrated in Figure 1. This proposition also establishes that the market breaks down when σ_N rises above σ_N^*.

The myopia of investors and the price insensitivity of noise traders jointly lead to this market breakdown. As a result of myopia, investors care only about the risk and return over the subsequent one period. As σ_N rises, investors demand a higher risk premium to take on a position against noise traders, i.e., a more positive coefficient p_N, which, in turn, leads to higher asset return volatility. Through this feedback process, once σ_N becomes larger than σ_N^*, the asset return volatility becomes so large that investors are unwilling to take any position regardless of the risk premium. If investors instead had longer horizons, they would be willing to take a position despite the return volatility over the short term, which would, in turn, stabilize the price impact of noise traders. The following proposition illustrates that, when investors have a two-period trading horizon, the market still breaks down if investors are sufficiently risk averse, while if they have infinite trading horizons, then a market equilibrium always exists.

Proposition 2 *If investors trade for two periods, then market failure occurs if their risk-aversion coefficient, γ, is sufficiently high. In contrast, when investors have an infinite horizon, a market equilibrium always exists.*

Proposition 2 highlights that it is the short-term horizon of investors that limits their capacity to trade against noise traders and contributes to market breakdown. This insight is reminiscent of the classic result highlighted by De Long et al. (1990), which shows that, in the presence of myopic arbitrageurs, noise traders can create their own space in asset prices even when there is no fundamental risk. In contrast to our setting, there is no market breakdown in De Long et al. (1990) because of the lack of fundamental risk in their model. Specifically, their model corresponds to a special case of our setting with $\sigma_D = \sigma_V = 0$. In this case, $\sigma_N^* \nearrow \infty$ according to (1), and consequently market breakdown never occurs. Without fundamental risk, rational investors recognize that any mispricing today will mean-revert in the next period. With fundamental risk the price may also move against them in the next period for fundamental reasons, which makes them more reluctant to trade against the price spread between the risky and riskless assets. The fundamental risk therefore gives rise to the market breakdown characterized by our model.[8] This mechanism for market breakdown is also distinct from that in Bhattacharya and Spiegel (1991), in which a monopolist insider, in the spirit of Kyle (1989), has an unbounded motive to manipulate its trading when uninformed investors respond excessively to information in prices.

The noise traders' price-insensitive trades serve to capture market rigidity that sometimes occurs as a result of either forced fire sales or panic selling during market turmoil. For example, Bian et al. (2017) document that the crash of China's stock market in the summer of 2015 was caused by the fire sales of highly leveraged stock investors. Our simple model describes a setting in which fire sales by some investors can lead a market breakdown because other investors are too short-termist to absorb these fire sales. This kind of market breakdown represents systemic failure and warrants government interventions.

3.4 Equilibrium with Government Intervention

We now incorporate government intervention into the model. Specifically, we augment the baseline setting to include a government that actively intervenes in the asset market. The government follows a linear trading rule:

$$X_t^G = -\vartheta_N N_t + \sigma_N \vartheta_N G_t.$$

[8] Market breakdown does not occur in Spiegel (1998), despite his model also including fundamental risk. As investors in Spiegel have infinite horizons, they are able to prevent market breakdown, even though multiple equilibria may still emerge. Watanabe (2008) analyzes a model with myopic investors and fundamental risk. Market breakdown would occur in his setting, but his analysis focuses on the implications of multiple equilibria, without even mentioning the possibility of market breakdown.

The first term $-\vartheta_N N_t$ captures the government's intended intervention strategy of trading against the noise traders, with the coefficient ϑ_N measuring the intensity of the intervention. We choose the convention of a negative coefficient because this term will partially offset noise trader demand when we later impose market clearing. We also include the second term $\sigma_N \vartheta_N G_t$ to capture unintended noise that arises from frictions in the intervention process, such as behavioral biases, lobbying effort, or information frictions. Specifically, $G_t = \sigma_G \varepsilon_t^G$ with $\varepsilon_t^G \sim \mathcal{N}(0,1)$ as independently and identically distributed shocks and σ_G as a volatility parameter. The magnitude of this noise component scales up with the intended intervention intensity $\sigma_N \vartheta_N$. This specification is reasonable as it is easier for frictions to affect the government's intervention when the intervention strategy requires more intensive trading. Furthermore, the government can neither correct nor trade against its own noise, because the noise originates from its own trading system. Instead, the government can internalize the amount of noise by choosing its trading intensity ϑ_N. For now, we take ϑ_N as given. We later specify a government objective in Section 5 to analyze its optimal intervention intensity choice.

Several notable features of our setting merit discussion. First, while we model government intervention as direct trading in asset markets, the specified intervention strategy captures the key feature of the Chinese government's broad-based policy interventions of leaning against cycles in the financial system, as summarized in Section 2. This simple linear intervention strategy allows us to take advantage of the well-developed framework from the market microstructure literature. Second, our specification of the government's intervention strategy is symmetric to both booms and busts. One may argue that, in practice, the government might be more concerned with preventing market crashes than market booms. To the extent that an unsustainable boom would eventually lead to a market crash, we believe it is reasonable to make the government equally concerned about mitigating both booms and crashes that are induced by noise traders.

As the government trades alongside investors to accommodate the trading of noise traders, the market-clearing condition $\int_0^1 X_t^i di + X_t^G + N_t = 0$ implies the following linear asset price function with the government noise as an additional factor:

$$P_t = \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_t + P_g G_t.$$

The following proposition rules out other nonlinear price equilibria and characterizes this linear market equilibrium, with the proof given in the Online Appendix.

Proposition 3 *For a given linear government intervention strategy with intensity ϑ_N, the asset market breaks down if*

$$\sigma_N > \frac{1}{\sqrt{\left(1-\vartheta_N\right)^2+\vartheta_N^2\sigma_G^2}}\sigma_N^*, \tag{2}$$

where σ_N^* is given in equation (1). Otherwise, an asset-market equilibrium exists, with price volatility increasing in σ_N^2 and price informativeness decreasing in σ_N^2.

As long as $\left(1-\vartheta_N\right)^2+\vartheta_N^2\sigma_G^2<1$, government intervention stabilizes the market by raising the critical value of noise-trader risk that induces breakdown. Note that $\left(1-\vartheta_N\right)^2+\vartheta_N^2\sigma_G^2$ is decreasing in ϑ_N for $\vartheta_N \in \left[0, \frac{1}{1+\sigma_G^2}\right]$ and is less than 1 for $\vartheta_N < 2/\left(1+\sigma_G^2\right)$. Thus, if $\vartheta_N < 2/\left(1+\sigma_G^2\right)$, the government's trading against noise traders makes the equilibrium existence condition slacker relative to the benchmark case without government intervention. The closer ϑ^N is to $\frac{1}{1+\sigma_G^2}$, the slacker is the equilibrium existence condition. This is shown in Figure 1, which depicts the shift in the market breakdown upper-bound and also the reduced asset price volatility before σ_N reaches the upper-bound. However, if $\vartheta_N > \frac{2}{1+\sigma_G^2}$, government trading actually raises the market breakdown upper-bound for σ_N due to the noise it injects into prices.

Taken together, government interventions in asset markets helps ensure market stability, especially during times of extreme market dysfunction when noise-trader risk is high. With informational frictions, however, the intervention to stabilize asset prices has additional effects on market dynamics, which we investigate in the next section.

4 An Extended Model with Information Frictions

We now extend the model to introduce realistic information frictions that investors and the government face in financial markets, while keeping the other features of the model the same as before. Specifically, we assume that the asset fundamental V_{t+1} and noise trading N_t are both unobservable at time t to all agents in the economy. For simplicity, we assume that the noise in government trading G_t is publicly observable at date t, albeit not before t. Since the government has no private information, this is equivalent to assuming that the scale of government intervention, X_t^G, is observable at date t.[9] As the government noise affects

[9] In an earlier draft of the paper, we analyzed the case with G_t being unobservable even after t. The results are qualitatively similar to our current setting, although the analysis is substantially more complex.

the asset price in equilibrium, investors have an incentive to acquire information about the next period's government noise. This extended model consequently allows us to analyze how government intervention interacts with both trading and information acquisition of investors, which ultimately affect information efficiency of asset prices.

4.1 Information and Equilibrium

We first describe the information structure of the economy and the asset-market equilibrium.

Public information. All market participants observe the full history of all public information, which includes all past dividends, asset prices, and government noise:

$$\mathcal{F}_t^M = \{D_s, P_s, G_s\}_{s \leq t},$$

which we will hereafter refer to as the "market" information set. We define

$$\hat{V}_{t+1}^M = E\left[V_{t+1} \mid \mathcal{F}_t^M\right]$$

as the conditional expectation of V_{t+1} with respect to $\mathcal{F}_t^M$. The government needs to trade against noise trading based on its conditional expectation of N_t. At the risk of abusing notation, we define

$$\hat{N}_t^M = E\left[N_t \mid \mathcal{F}_t^M\right],$$

which represents expectation of the current-period N_t rather than N_{t+1}. We also define

$$\hat{G}_{t+1}^M = E\left[G_{t+1} \mid \mathcal{F}_t^M\right]$$

as the market's conditional expectation of the next-period G_{t+1}. These three belief variables, $\hat{V}_{t+1}^M$, $\hat{N}_t^M$, and $\hat{G}_{t+1}^M$, are time-t expectations of V_{t+1}, N_t, and G_{t+1}, respectively. Together with the publicly observed current-period G_t, they summarize the public information at time t regarding the aggregate state of the market. We collect these variables as a state vector:

$$\Psi_t = \begin{bmatrix} \hat{V}_{t+1}^M & \hat{N}_t^M & \hat{G}_{t+1}^M & G_t \end{bmatrix}.$$

Government intervention. We assume that the government does not have any private information. Instead, at date t the government trades against noise traders based only on the publicly available information $\mathcal{F}_t^M$.[10] As before, we adopt the following intervention program, instituted to trade against the conditional market expectation $\hat{N}_t^M$:

$$X_t^G = -\vartheta_{\hat{N}} \hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}} \hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]} G_t, \tag{3}$$

where $\vartheta_{\hat{N}}$ is the intensity of the government's intervention. We also extend the noise brought by the government intervention to be increasing with the conditional variance of government trading, $\sqrt{Var\left[\vartheta_{\hat{N}} \hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}$, which is consistent with $\sigma_N \vartheta_N$ in the perfect-information case. In this section, we continue to take the government's intervention intensity $\vartheta_{\hat{N}}$ as given and focus on analyzing investors' information choice. We will analyze the government's intervention choice in the next section.

Investors' information choice. In each period, investors face uncertainty in the asset fundamental, the noise trading, and the government noise. Specifically, at date t, each investor can choose to acquire a private signal about either the next-period asset fundamental V_{t+1} or the next-period government noise G_{t+1}. We denote the investor's choice as $a_t^i \in \{0, 1\}$, with 1 representing the choice of a fundamental signal and 0 the choice of a signal about the government noise.[11] When the investor chooses $a_t^i = 1$, the fundamental signal is

$$s_t^i = V_{t+1} + 1/\sqrt{a_t^i \tau_s} \varepsilon_t^{s,i},$$

where $\varepsilon_t^{s,i} \sim \mathcal{N}(0, 1)$ is signal noise, independent of all other random variables in the setting, and τ_s represents the precision of the signal if chosen. When the investor chooses $a_t^i = 0$, the government signal is

$$g_t^i = G_{t+1} + 1/\sqrt{\left(1 - a_t^i\right) \tau_g} \varepsilon_t^{g,i},$$

[10] In a previous draft, we adopted an alternative setting in which the government possesses private signals about the fundamental. This private information causes the government to hold different beliefs about the fundamental and noise trading from investors and, more importantly, makes the government's trading not fully observable to the investors. Through this latter channel, the noise in the government's signals endogenizes the government's intervention noise G_t. Such a structure substantially complicates the analysis by introducing a double learning problem for the investors to acquire information about the government's belief, which is itself the outcome of a learning process. It is reassuring that this more elaborate setting gives similar results as in our current setting with exogenous government intervention noise.

[11] Generally speaking, the investors may also acquire private information about noise trading, rather than asset fundamental and government noise. Introducing such a third type of private information would complicate the analysis without any particular gain in economic insight. In our current setting, each investor can indirectly infer the value of noise trading through the publicly observed asset price. See, for instance, Ganguli and Yang (2009) for a setting in which investors can learn either about the asset fundamental or noise trading.

where $\varepsilon_t^{g,i} \sim \mathcal{N}(0,1)$ is signal noise, independent of all other random variables in the setting, and τ_g represents the precision of the signal if chosen. These signals allow the investor to better predict the next-period asset return by forming more precise beliefs about V_{t+1} and G_{t+1}. Motivated by limited investor attention and a realistic fixed cost in information acquisition, we assume that each investor chooses one and only one of these two signals.[12]

At date t, each investor first makes his information acquisition choice a_t^i based on the public information set $\mathcal{F}_{t-1}^M$ from the previous period. After receiving his private information $a_t^i s_t^i + (1-a_t^i)\, g_t^i$ and the public information D_t, P_t, and G_t released during the period, the investor chooses his asset position X_t^i to maximize his expected utility:

$$U_t^i = \max_{a_t^i \in \{0,1\}} E\left[\max_{X_t^i} E\left[-\exp\left(-\gamma W_{t+1}^i\right) \mid \mathcal{F}_t^i \right] \middle| \, \mathcal{F}_{t-1}^M \right],$$

where the investor's full information set $\mathcal{F}_t^i$ is

$$\mathcal{F}_t^i = \mathcal{F}_t^M \vee \left\{ a_t^i, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i \right\}.$$

Noisy rational expectations equilibrium. Market clearing of the asset market requires that the net demand from the investors and the government be equal to the supply of the noise traders at each date t: $\int_0^1 X_t^i di + X_t^G + N_t = 0$. By assuming elastic supply of riskless debt, the credit market clears automatically.

We also assume that the investors and the government have an initial prior with Gaussian distributions at $t=0$: $(V_0, N_0) \sim \mathcal{N}\left(\left(\bar{V}, \bar{N}\right), \Sigma_0\right)$, where $\Sigma_0 = \begin{bmatrix} \Sigma_0^V & 0 \\ 0 & \Sigma_0^N \end{bmatrix}$. Note that the variables in both $\mathcal{F}_t^M$ and $\mathcal{F}_t^i$ all have Gaussian distributions. As a result, conditional beliefs of the investors and the government about V_t and N_t under any of the information sets are always Gaussian. Furthermore, the variances of these conditional beliefs follow deterministic dynamics over time and will converge to their respective steady-state levels at exponential rates. Throughout our analysis, we will focus on steady-state equilibria, in which the belief variances of the government and investors have reached their respective steady-state levels and their policies are time homogeneous.

[12] Instead of a discrete information acquisition choice $a \in \{0,1\}$, one could generalize our framework to allow for a continuous choice $a \in [0,1]$, which corresponds to a signal that is partially informative about both the fundamental and the government noise. We conjecture that, in such a setting, instead of having a government-centric outcome, investors would nevertheless tilt their information acquisition too much toward acquiring government information, when the government's objective is to minimize price volatility.

At time t, a Noisy rational expectations equilibrium is a list of policy functions: $a^i(\Psi_{t-1})$, and $X^i(\Psi_t, a_t^i, a_t^i s_t^i + (1-a_t^i) g_t^i, P_t)$, and a price function $P(\Psi_t, V_{t+1}, N_t, G_{t+1})$, which jointly satisfy the following:

- Investor optimization: each investor i takes as given the government's intervention strategy $\vartheta_{\hat{N}}$ to make his information acquisition choice $a_t^i = a^i(\Psi_{t-1})$ based on his ex ante information set $\mathcal{F}_{t-1}^M$ and then makes his investment choice $X^i(\Psi_t, a_t^i, a_t^i s_t^i + (1-a_t^i) g_t^i, P_t)$ based on other investors' information acquisition choices $\left\{a_t^{-i}\right\}_{-i}$ and his full information set $\mathcal{F}_t^i$.
- Market clearing:

$$\int_0^1 X^i\left(\Psi_t, a_t^i, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i, P_t\right) di + X^G(\Psi_t) + N_t = 0.$$

- Consistency: investor i and the government form their expectations of V_{t+1}, G_{t+1}, and N_t based on their information sets $\mathcal{F}_t^i$ and $\mathcal{F}_t^M$, respectively, according to Bayes' Rule.

4.2 The Equilibrium

We now analyze the equilibrium by describing its key elements to convey the key economic mechanism of the model. The complete steps of deriving the equilibrium and formulas are in Appendix A.

4.2.1 Price Conjecture and Equilibrium Beliefs

With government intervention introducing noise into the equilibrium asset price as an additional factor, each investor faces a nontrivial choice at date t in whether to acquire private information about either the next-period fundamental V_{t+1} or government noise G_{t+1}. When all investors choose to acquire information about the government noise, the asset price does not aggregate any private information about V_{t+1} but rather brings the next-period government noise G_{t+1} into the current-period asset price. To analyze the equilibrium asset price, we begin by conjecturing a linear price function:[13]

$$P_t = \frac{1}{R^f - \rho_V}\hat{V}_{t+1}^M + p_g G_t + p_{\hat{G}}\hat{G}_{t+1}^M + p_V\left(V_{t+1} - \hat{V}_{t+1}^M\right) + p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right) + p_N N_t. \quad (4)$$

[13] This conjectured functional form is not unique because the market's beliefs about V_{t+1}, N_t, and G_{t+1} are correlated objects after observing the asset price. That is, $\hat{N}_t^M$ can be replaced by a linear combination of P_t, $\hat{V}_{t+1}^M$, and $\hat{G}_{t+1}^M$ and as such does not have to appear in the price function, even though $\hat{N}_t^M$ determines the government's intervention.

The first term $\frac{1}{R^f - \rho_V}\hat{V}_{t+1}^M$ is the expected asset fundamental conditional on the market information $\mathcal{F}_t^M$ at date t, the term $p_g G_t$ reflects the noise introduced by the government into the asset demand in the current period, while the term $p_{\hat{G}}\hat{G}_{t+1}^M$ reflects the market expectation of the government noise in the next period. These three pieces serve as anchors in the asset price based on the public information. The fourth term $p_V\left(V_{t+1} - \hat{V}_{t+1}^M\right)$ captures the fundamental information aggregated through the investors' trading. Following the insight from Hellwig (1980), if each investor acquires a private signal about the asset fundamental V_{t+1}, their trading aggregates their private signals and allows the equilibrium price to partially reveal V_{t+1}. If all investors choose to acquire information about the next-period government noise G_{t+1}, instead of V_{t+1}, the coefficient of this term p_V would be zero. Instead, their trading aggregates their private information about G_{t+1}, as captured by the fifth term $p_G\left(G_{t+1} - \hat{G}_{t+1}^M\right)$.[14] The final term $p_N N_t$ represents the price impact of noise trading.

Given the asset price in (4), in order to predict the asset return, an individual investor needs to infer not only the asset fundamental, V_{t+1}, but also the government noise, G_{t+1}. As each individual investor has a piece of a private signal, $a_t^i s_t^i + (1 - a_t^i)\ g_t^i$, his learning process simply requires adding this additional signal to the market beliefs. We summarize the filtering process through the updating equation as

$$\begin{aligned}\begin{bmatrix}\hat{V}_{t+1}^i \\ \hat{G}_{t+1}^i\end{bmatrix} &= \begin{bmatrix}\hat{V}_{t+1}^M \\ \hat{G}_{t+1}^M\end{bmatrix} + CoV\left\{\begin{bmatrix}V_{t+1} \\ G_{t+1}\end{bmatrix}, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \middle|\ \mathcal{F}_t^M\right\} \\ &\quad \cdot Var\left\{a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i \middle|\ \mathcal{F}_t^M\right\}^{-1}\left[a_t^i\left(s_t^i - \hat{V}_{t+1}^M\right) + \left(1 - a_t^i\right)\left(g_t^i - \hat{G}_{t+1}^M\right)\right].\end{aligned}$$

The variance and covariance in this expression depend on various endogenous objects such as the informativeness of the equilibrium asset price and the precision of the market beliefs, and fully derived in Appendix A. This expression makes clear that the investor's private signal helps him infer the asset fundamental or the government's trading noise in the next period, both of which impact the asset return.

4.2.2 Information Choice

To analyze an individual investor's information choice, it is convenient to decompose the expected asset return based on his information set relative to the market information set.

[14] There is no need to incorporate a term related to investors' (higher order) cross-beliefs about V_{t+1} or G_{t+1} because $\int_0^1 a_t^i s_t^i di = V_{t+1}$ and $\int_0^1 \left(1 - a_t^i\right) g_t^i di = G_{t+1}$ by the Weak Law of Large Numbers.

We can update $E\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from $E\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ by the Bayes' Rule according to

$$
\begin{aligned}
& E\left[R_{t+1} \mid \mathcal{F}_t^i\right] = E\left[R_{t+1} \mid \mathcal{F}_t^M \vee a_t^i s_t^i + \left(1-a_t^i\right) g_t^i\right] \\
= \ & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{CoV\left[R_{t+1}, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i \mid \ \mathcal{F}_t^M\right]}{Var\left[a_t^i s_t^i + \left(1-a_t^i\right) g_t^i \mid \ \mathcal{F}_t^M\right]} \cdot \left[a_t^i \left(s_t^i - \hat{V}_{t+1}^M\right) + \left(1-a_t^i\right)\left(g_t^i - \hat{G}_{t+1}^M\right)\right].
\end{aligned}
$$

The investor's private information through either s_t^i or g_t^i helps him better predict the excess asset return relative to the market information. Given the investor's myopic CARA preferences, his demand for the asset is

$$
X^i = \frac{1}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}. \tag{5}
$$

In choosing whether to acquire either s_t^i or g_t^i at date t, the investor maximizes his expected utility based on the ex ante market information:

$$
E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = \max_{a_t^i \in \{0,1\}} -E\left\{E\left[\exp\left(-\gamma R^f \bar{W} - \frac{1}{2} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\right) \middle| \mathcal{F}_t^M\right] \middle| \mathcal{F}_{t-1}^M\right\},
$$

which has already incorporated the investor's optimal asset position in (5).

The investor's expected CARA utility in our Gaussian framework is fully determined by the second moment of the return distribution conditional on his information set $\mathcal{F}_t^i$. This nice feature allows us to simplify his information choice to

$$
a_t^i = \arg\max_{a_t^i \in \{0,1\}} -Var\left[R_{t+1} | \mathcal{F}_t^M, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i, a_t^i\right].
$$

This objective involves only minimizing the conditional price change variance, which is stationary in the steady-state equilibria that we consider. Thus, the information acquisition choice faced by each individual investor is time invariant. By noting that

$$
Var\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i\right] = Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] - \frac{CoV\left[R_{t+1}, a_t^i s_t^i + \left(1-a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[a_t^i s_t^i + \left(1-a_t^i\right) g_t^i \mid \mathcal{F}_t^M\right]},
$$

we arrive at the following proposition, which corresponds to Proposition A7 in Appendix A.

Proposition 4 *At date t, investor i chooses to acquire information about the next-period fundamental V_{t+1} if $\frac{CoV\left[R_{t+1}, g_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[g_t^i \mid \mathcal{F}_t^M\right]} < \frac{CoV\left[R_{t+1}, s_t^i \mid \mathcal{F}_t^M\right]^2}{Var\left[s_t^i \mid \mathcal{F}_t^M\right]}$ and about the next-period government noise G_{t+1} otherwise.*

The investor chooses his signal to maximize his informational advantage over the public information set when trading. Proposition 4 states that this objective is equivalent to choosing the signal that leads to a greater reduction in the conditional variance of the excess asset return. The investor may choose to acquire the signal on the government noise over the signal on the asset fundamental, because the government noise affects the asset return when the investor sells his asset holding on the next date. As a result, the more the government noise covaries with the unpredictable component of the asset return from the market information set, the more valuable the signal about the government noise is to the investor.

In models of information aggregation, such as in Grossman and Stiglitz (1980) and Hellwig (1980), information choices among investors are typically strategic substitutes. That is, all else equal, if some investors at time t acquire private information about V_{t+1}, then the equilibrium asset price at time t will become more informative about it, and this *reduces* the incentives of other investors to acquire information about V_{t+1}. In models in which investors can acquire different sources of information, including those in Ganguli and Yang (2009), Manzano and Vives (2011), and Farboodi and Veldkamp (2016), information choices can exhibit intratemporal strategic complementarity. As some investors learn more about one source of information, asset prices become more informative of the fundamentals, *strengthening* the incentive of other investors to acquire information, albeit about a different source.

Interestingly, our model features intertemporal complementarity between investors' information choices and government policy across periods. Similar, for instance, to Froot, Scharfstein, and Stein (1992), investors have incentive to align their information choices across generations when the asset fundamental is persistent.[15] If more investors at time $t+1$ acquire information about V_{t+2}, then there is greater incentive for investors at time t to acquire information about V_{t+1}, as V_{t+2} partially reflects V_{t+1}. Novel to our setting, however, is that there is also intertemporal complementarity between the government's announced intervention policy at time $t+1$ and investors' choice to learn about G_{t+1} at date t, since the government is a large trader with price impact. Importantly, the government internalizes that it can influence the investors' information choices when choosing its policy.[16] In contrast to Hellwig and Veldkamp (2009), in which intratemporal complementarity in agents'

[15] This intertemporal complementarity does not operate through G_t, since it is independent over time. If we were to relax this simplifying assumption, as we did in a previous version of the paper, the model will display even stronger complementarity in investors' information choices.

[16] This is also in contrast to the literature on information aggregation with strategic traders, as in, for instance, Kyle (1989). Since the solution concept in these models is an "equilibrium in demand curves," large traders do not internalize that they can impact the learning and information decisions of other large traders. As such, these equilibria are ex post efficient up to the impact of market power.

actions leads to complementarity in their information choices, here the government's future intervention policy induces investors today to learn about future noise in government intervention, since the government's policy materially impacts their return from trading the risky asset. This complementarity can be sufficiently strong to dominate the substitution effect in information choice across investors, and to cause all of them to acquire private information about the same variable.

The choice of an individual investor to acquire information about the government noise rather than the asset fundamental introduces an externality for the overall market. When investors devote their limited attention to do so, less information about the asset fundamental is imputed into the asset price, which causes the asset price to be a poorer signal about the asset fundamental. In addition, as investors devote attention to learning about G_{t+1}, the asset price will aggregate more of the investors' private information about G_{t+1}, causing the next-period government noise to impact the current-period asset price. In this sense, the investors' speculation of government noise may exacerbate its impact on asset prices.

4.2.3 Market Equilibrium

Given the investors' optimal information and asset choices and the government's intervention strategy, we have the following market-clearing condition:

$$\begin{aligned} 0 &= N_t - \vartheta_{\hat{N}} \hat{N}_t^M + \sqrt{Var\left[\vartheta_{\hat{N}} \hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]} G_t + \int \frac{a_t^i}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, s_t^i\right]} di \\ &\quad + \int \frac{1-a_t^i}{\gamma} \frac{E\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^M, g_t^i\right]} di. \end{aligned}$$

The Weak Law of Large Numbers implies that aggregating the investors' asset positions will partially reveal their private information about V_{t+1} if $\int a_t^i di = 1$ and G_{t+1} if $\int a_t^i di = 0$. By matching the coefficients of all the terms on both sides of this equation, we obtain a set of equations to determine the coefficients of the conjectured equilibrium price function in (4).

There can exist several types of equilibrium.

- **Fundamental-centric outcome**. When all investors choose to acquire information about the asset fundamental, the asset price aggregates the investors' private information and partially reflects the asset fundamental, but does not reflect the next-period government noise. As a result, the asset price takes a particular form of

$$P_t = \frac{1}{R^f - \rho_V} \hat{V}_{t+1}^M + p_g G_t + p_V \left(V_{t+1} - \hat{V}_{t+1}^M\right) + p_N N_t, \qquad (6)$$

which is different from the general asset price specification in (4) in that the terms $p_{\hat{G}}\hat{G}^M_{t+1}$ and $p_G\left(G_{t+1}-\hat{G}^M_{t+1}\right)$ do not appear.

- **Government-centric outcome**. When all investors choose to acquire information about the next-period government noise, the asset price partially reflects the next-period government noise but not the asset fundamental:

$$P_t = \frac{1}{R^f-\rho_V}\hat{V}^M_{t+1} + p_g G_t + p_{\hat{G}}\hat{G}^M_{t+1} + p_G\left(G_{t+1}-\hat{G}^M_{t+1}\right) + p_N N_t,$$

  where the term $p_V\left(V_{t+1}-\hat{V}^M_{t+1}\right)$ does not appear.

- **Mixed outcome**. It is also possible to have a mixed equilibrium with a fraction of the investors acquiring information about the asset fundamental and the others having information about the government noise. In such a mixed equilibrium, the general price function specified in (4) prevails.

Depending on the model parameters, all three types of equilibrium may appear. In the special case that the fundamental V_t is i.i.d., or $\rho_V=0$, the following proposition establishes a necessary and sufficient condition for the government-centric equilibrium to occur for a given government intervention intensity $\vartheta_{\hat{N}}$.

Proposition 5 *Suppose $\rho_V=0$, and fix a government intervention intensity $\vartheta_{\hat{N}}$. A government-centric equilibrium exists under a necessary and sufficient condition:*

$$\begin{aligned}&\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}} - \sqrt{\left(\frac{1}{2\sigma_N c}\frac{R^f}{1-\vartheta_{\hat{N}}}\right)^2 - \frac{\sigma_V^2+\sigma_D^2}{c}} \\ \geq\ & \frac{\sigma_V^2}{\sqrt{\sigma_V^2+\tau_s^{-1}}}(1+x)\sqrt{\left(\sigma_G^2+(1+x)\tau_g^{-1}\right)\left(\frac{\frac{1-\vartheta_{\hat{N}}}{\vartheta_{\hat{N}}}}{\sigma_G^2 - R^f\frac{x}{1-\vartheta_{\hat{N}}}}\right)^2},\end{aligned} \tag{7}$$

where x is given by

$$x(1+x)^3 = \left(\frac{\vartheta_{\hat{N}}}{R^f}\sigma_G^3\right)^2,$$

and c is a nonnegative function of $\left\{\vartheta_{\hat{N}}, R^f, \sigma_G\right\}$ given in the Online Appendix. This equilibrium is more likely to exist the higher are σ_N^2 and σ_D^2, and it always exists for σ_V^2 that is sufficiently small.

Table I: Baseline Model Parameters

Government:	$\gamma_\sigma = 1.25,\ \gamma_V = 1,\ \sigma_G^2 = 2$
Asset Fundamental:	$\rho_V = 0.75,\ \sigma_V^2 = 0.01,\ \sigma_D^2 = .8$
Noise Trading:	$\sigma_N^2 = 0.2$
Investors:	$\gamma = 1,\ \tau_s = 500,\ \tau_g = 500,\ R^f = 1.01$

In a government-centric equilibrium, the asset price P_t aggregates only private information about the future noise in government trading, G_{t+1}. In this situation, all investors are willing to acquire information about G_{t+1} if it reduces their conditional uncertainty about the future price, P_{t+1}, which contains G_{t+1} through the government's trading, more than would learning about the fundamental, V_{t+1}. Proposition 5 reveals that this can occur for two reasons. The first is that the benefit to learning about the fundamental, as measured by its uncertainty, σ_V^2, is small. The second is that the benefit to learning about the future noise in the government's trading is large. The larger the noise in prices from noise trading, $p_N \sigma_N$ (which is the left-hand side of (7)), the less aggregated private information about G_{t+1} is revealed by the price, and the more motivated investors are to acquire private information about G_{t+1}. Since $p_N \sigma_N$ is increasing in the uncertainty about noise trading and the unlearnable part of the dividend, σ_N^2 and σ_D^2, respectively, a government-centric equilibrium is more likely to occur the larger are σ_N^2 and σ_D^2.

4.3 Effects of Government Intervention

This subsection analyzes how government intervention affects the market dynamics. For comparison, we also include a benchmark case without government intervention, which corresponds to the classic Hellwig (1980) equilibrium, in which each investor acquires a fundamental signal, and the equilibrium asset price follows the form in (6). Proposition A1 in the Appendix characterizes the Hellwig equilibrium and, in particular, shows that information frictions reduce the critical level of noise-trader risk so that the market is more likely to break down. Proposition A2 further shows that even if an equilibrium exists, asset price volatility is higher and price efficiency is lower in the presence of information frictions.

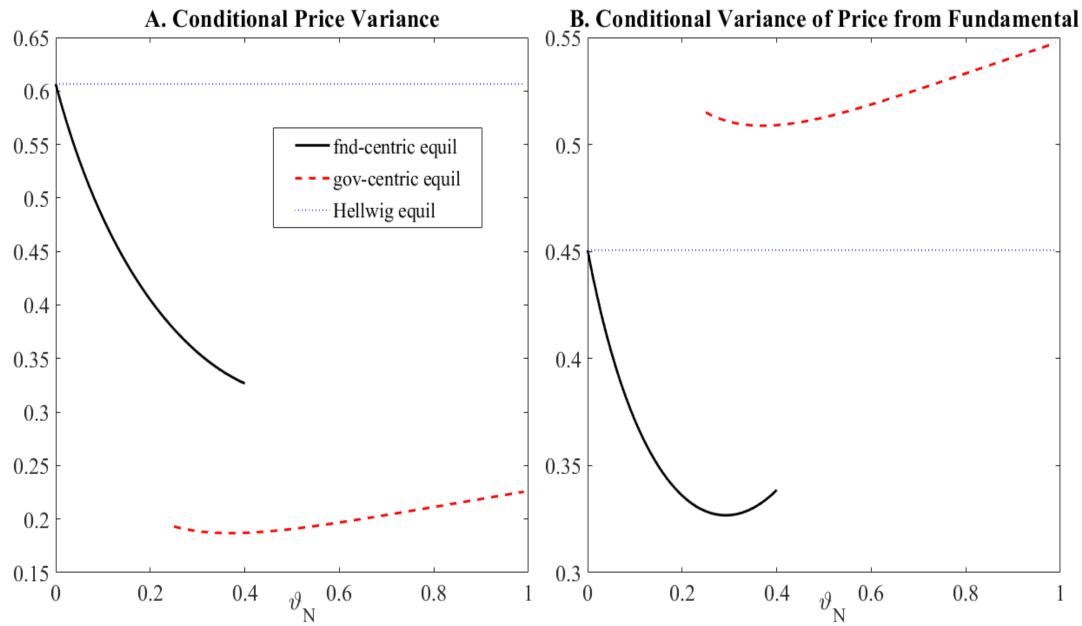

Figure 2: Equilibrium dynamics across intervention intensity $\vartheta_{\hat{N}}$. Panel A depicts the conditional price variance and Panel B the conditional variance of price deviation from the fundamental.

We analyze the effects of government intervention through a series of numerical examples, based on a set of baseline parameter values listed in Table I. Figure 2 illustrates how the asset market dynamics vary with a given intensity $\vartheta_{\hat{N}}$ of the government intervention. As we will discuss in the next section, the government can choose an optimal level of intervention intensity to accomplish a certain policy objective. Panels A and B depict the conditional asset price variance $Var\left[\Delta P_t\left(\vartheta_{\hat{N}}\right)|\mathcal{F}_{t-1}^M\right]$ and the conditional asset price deviation from fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_V}V_{t+1}\,|\mathcal{F}_{t-1}^M\right]$, our price efficiency measures, respectively.

As the government gradually increases its intervention intensity $\vartheta_{\hat{N}}$ from zero, investors continue to acquire information about the fundamental. In this fundamental-centric equilibrium, both conditional price variance and conditional price deviation from fundamental drop from their respective values in the Hellwig benchmark, confirming the common wisdom that, by leaning against noise traders, government intervention ensures financial stability and improves price efficiency.

More interestingly, Figure 2 shows that by trading more aggressively against noise traders, ensuring financial stability and improving price efficiency are not always consistent with each other, which is a key insight of our model. Specifically, as $\vartheta_{\hat{N}}$ exceeds 0.22, a government-centric equilibrium emerges with all of the investors choosing to acquire information about the government noise. When the market transitions from the fundamental-centric equilibrium to the government-centric equilibrium, the asset price variance slumps downward, indicating that government intervention is able to further mitigate the price effect of noise traders. The conditional variance of the price deviation from its fundamental value jumps up, however, suggesting that price efficiency is reduced rather than improved. This occurs because intensive government intervention makes government noise an important factor in asset returns, which in turn diverts investor attention from acquiring fundamental information to acquiring information about future government noise. Panel B shows that when this happens, price efficiency can become even worse than the benchmark case without government intervention.

Figure 2 also shows a more subtle implication of our model: the government-centric equilibrium may allow the government to more effectively reduce the price impact of noise traders without trading more. When the intervention intensity $\vartheta_{\hat{N}}$ is in an intermediate range between 0.22 and 0.40, both the fundamental-centric and the government-centric equilibria exist,[17] as a result of the aforementioned intertemporal complementarity in investors' information choices.[18] Comparing these two equilibria for a given level of intervention intensity shows that asset price volatility is substantially lower in the government-centric equilibrium without requiring more government trading. This happens because, in the fundamental-centric equilibrium, each investor has his own private information about the asset fundamental, and the private information causes investors to hold beliefs different from each other and from the government about not only the asset fundamental but also the current-period noise trading. As a result, the government has to trade against not only noise traders but also investors. Investors' trading disseminates their private fundamental information into the asset price and improves its information efficiency, but partially offsets the government's effort to counter noise traders. In contrast, in the government-centric equilibrium, investors' private information is about the next-period government noise, and, like the government, investors all use the same public information to infer the current-period noise trading. Consequently, investors tend to trade against noise traders along the same direction as the government, thereby reinforcing the effectiveness of the government's intervention in reducing

[17] A mixing equilibrium is also possible when $\vartheta_{\hat{N}}$ is in this range. For simplicity, we omit discussions of the mixed equilibrium. In the presence of the multiple equilibria, we assume that the government, as a large player in the game, has the capacity to select the equilibrium most desirable to its objective.

[18] The presence of this strong intertemporal complementarity also implies that even if each investor is free to choose a mixed signal that is partially informative about the asset fundamental and the government noise (as discussed in Footnote 12), the investor may nevertheless choose to focus on a pure signal about either the asset fundamental or the government noise.

volatility. This mechanism further highlights the tension between reducing price volatility and improving price efficiency.

5 Intervention Objective

In this section, we discuss the objective of government intervention and analyze the resulting optimal intervention intensity. We first discuss two intervention objectives, one to reduce price volatility and the other to improve price efficiency, and then expand the model setting to provide a full welfare analysis of government intervention. In our analysis, we assume that the government, as a large player in the market, has the capacity to choose an equilibrium aligned closest to its objective in the presence of multiple equilibria among investors.

5.1 Reducing Volatility versus Improving Efficiency

This subsection discusses two widely recognized intervention objectives by policy makers. One is to minimize asset price volatility, which is equivalent to reducing $Var\left[\Delta P_t\left(\vartheta_{\hat{N},t}\right)|\mathcal{F}_{t-1}^M\right]$, the conditional asset price variance, and the other is to improve asset price efficiency, which is equivalent to reducing $Var\left[P_t\left(\vartheta_{\hat{N},t}\right)-\frac{1}{R^f-\rho_V}V_{t+1}\ |\mathcal{F}_{t-1}^M\right]$, the conditional variance of the deviation of the asset price from its fundamental value. Reducing price volatility is consistent with attenuating the risk premia required by market participants and the destabilizing effects of asset price volatility on leveraged investors and firms, thereby ensuring financial stability as suggested by Brunnermeier and Pedersen (2009) and Geanakoplos (2010). Improving asset price efficiency is consistent with making asset prices more informative, and consequently more efficient in guiding resource allocation in the economy, as reviewed by Bond, Edmans, and Goldstein (2012). These two objectives are often viewed as congruent with each other, as an intervention strategy of leaning against noise trading reduces the impact of noise trading on asset prices, thus reducing price volatility and improving price efficiency. With price volatility being much easier to measure in practice than price efficiency, policy makers tend to use reducing price volatility as an operational intervention objective; see, e.g., Stein and Sundarem (2018).

For simplicity of exposition, we do not impose any budget constraint on the government's intervention. We will consider the government's trading profit in our welfare analysis in the next subsection. Interestingly, despite the absence of any budget constraint, there is an interior optimum to the government's intervention strategy because it internalizes the

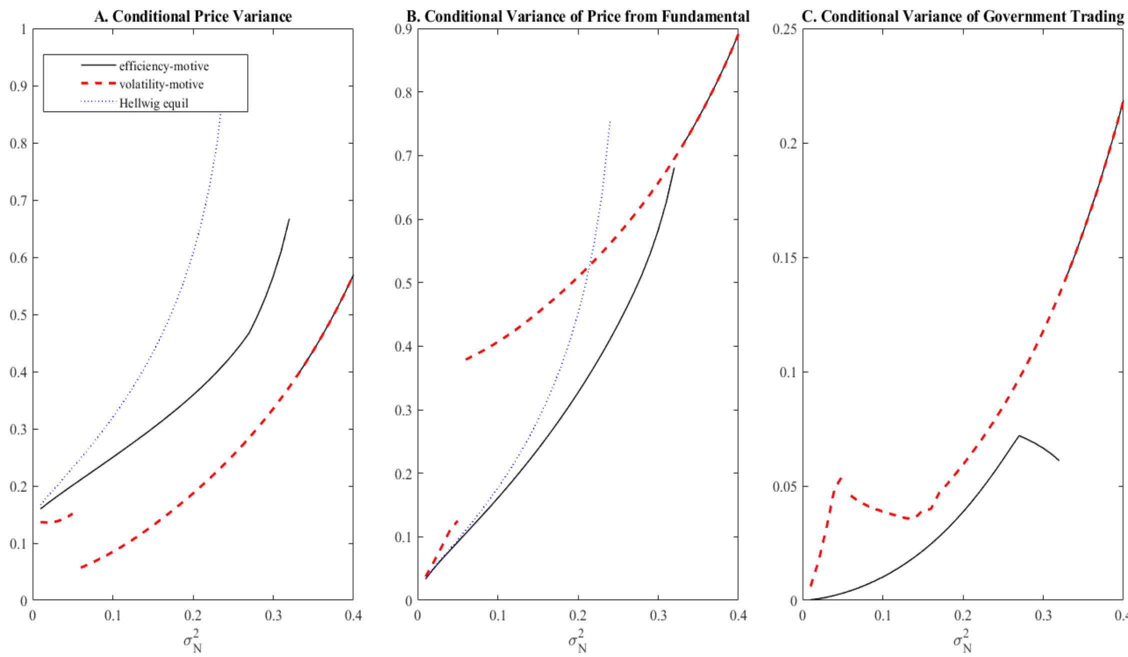

Figure 3: Equilibrium dynamics across noise-trader risk. Panel A depicts the conditional price variance $Var\left[P_t\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-1}^M\right]$, Panel B the conditional variance of price deviation from the fundamental $Var\left[P_t\left(\vartheta_{\hat{N}}\right) - \frac{1}{R^f-\rho_v}v_{t+1} \mid \mathcal{F}_{t-1}^M\right]$, and Panel C the conditional variance of government trading. In these panels, the dotted line represents the Hellwig equilibrium without government intervention, the solid line the equilibrium with government intervention to improve price efficiency, and the dashed line the equilibrium with government intervention to reduce price volatility.

amount of noise that its intervention introduces into the market. As illustrated in Figure 2, an excessive intervention intensity $\vartheta_{\hat{N}}$ succeeds more in introducing additional government noise into the asset price than it does in removing noise trading.

Figure 3 highlights an important observation: the objectives of reducing volatility and improving efficiency can lead to sharply different equilibrium dynamics. Specifically, Figure 3 depicts conditional price variance in Panel A, conditional variance of the deviation of the price from its fundamental value in Panel B, and conditional variance of government trading in Panel C across different values of noise-trading variance σ_N^2 for three equilibria: 1) the Hellwig equilibrium without government intervention (the dotted line), 2) government intervention with an objective to improve price efficiency (the solid line), and 3) government intervention with an objective to reduce price volatility (the dashed line).

With the objective to improve price efficiency, the optimal intervention policy attempts to remain in the fundamental-centric equilibrium when σ_N^2 is below a level of around 0.33 and only jumps to the government-centric equilibrium for σ_N^2 above that. The optimal intervention intensity is typically increasing in noise-trading risk. An exception is the region of σ_N^2 between 0.27 and 0.33. In this region, the government needs to reduce its intervention coefficient in order to stay in the fundamental-centric equilibrium. Consequently, while the conditional price variance and conditional price deviation from fundamental are both increasing with noise-trading risk, their levels are lower than the respective levels in the Hellwig equilibrium without government intervention.

If the objective is to reduce price volatility, the optimal intervention policy generally entails a higher level of government trading intensity relative to that under the objective of improving efficiency. The more intensive intervention causes the market to shift from a fundamental-centric equilibrium to a government-centric equilibrium at a substantially lower level of σ_N^2 around 0.05, rather than 0.33. The intervention leads to lower price volatility, but worse price efficiency. Surprisingly, the price efficiency is even worse than that in the Hellwig equilibrium without government intervention when σ_N^2 is below a level of around 0.22. When σ_N^2 is above 0.05, intensive government intervention distracts investors from acquiring information about the asset fundamental and instead focuses their attention on future government noise. This is the key mechanism that leads to the reduced price efficiency.[19]

Figure 3 illustrates that, in contrast to the common practice of treating volatility reduction as equivalent to improving price efficiency, these objectives can lead to sharply different intervention intensities and market dynamics. These differences are due to the impact of government intervention on investors' information acquisition. To highlight this channel, Proposition 6 formally shows that, fixing investors' information acquisition in a government-centric equilibrium, these two objectives are consistent with each other.

Proposition 6 *In a government-centric equilibrium, improving price efficiency is equivalent to reducing price volatility.*

5.2 Welfare Analysis

We now provide a welfare analysis of government intervention by expanding the model setting to include four groups of agents: investors, noise traders, entrepreneurs, and taxpayers. For simplicity, we assume that these four groups do not overlap. All agents are risk averse and

[19] When σ_N^2 is below 0.05, the market stays in a fundamental-centric equilibrium with investors acquiring fundamental information. Price efficiency is nevertheless worse than with the Hellwig equilibrium because of the noise generated by the government's more intensive intervention.

have CARA utility with a common coefficient of absolute risk aversion γ. In order not to overload the paper, we relegate the full model setting to Appendix B and provide only a brief introduction of the four groups here:

- The first group, investors, follows directly from the main model in Section 4, and their expected utility in each period is derived in (A7).
- We microfound noise traders as discretionary liquidity traders, in a way similar to Han, Tang, and Yang (2016), in order to explicitly account for their welfare from trading. These liquidity traders participate in asset-market trading to receive a hedging benefit by submitting a market order of random size in each period. We derive their expected utility in each period in (A8).
- We also introduce a group of entrepreneurs, who can invest in risky projects whose payoffs are correlated with the traded asset. As a result, these entrepreneurs benefit from extracting useful information from the asset price. We show in (A9) that their expected utility is decreasing with $\Sigma^{M,VV}$—an inverse measure of the asset price efficiency (or, specifically, the conditional variance of the asset fundamental based on each period's public information)—and σ_y^2, the variance of project-specific noise. Also, as project-specific noise, σ_y^2, rises, the usefulness of the asset price signal, i.e. the impact of $\Sigma^{M,VV}$ on entrepreneurs' welfare, declines.
- We also include a fourth group, taxpayers, as the residual claimants to the government's trading profit. Their expected utility from the government's trading profit in each period is given in (A10).

We assume that the government maximizes the Nash social welfare function proposed by Kaneko and Nakamura (1979), which is a monotonic transformation of the product of the utilities of all agents in the economy. As specified in (A11), this welfare function is essentially given by the sum of the logarithmic expected utilities of the four aforementioned groups. As each group has CARA utility and Gaussian-distributed payoffs, its logarithmic expected utility is the sum of its expected profit and a utility penalty for risk that is decreasing in the conditional payoff variance. As the asset-market trading is a zero-sum game among investors, liquidity traders, and taxpayers, we are able to establish the following proposition for the objective function of government intervention, which is fully determined by the second-order moments of the asset return:

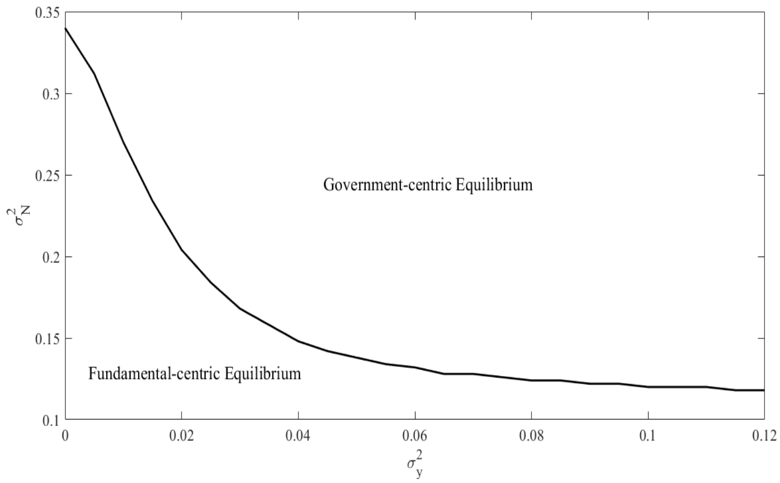

Figure 4: Boundary between fundamental-centric and government-centric equilibria based on the baseline parameter values listed in Table I.

Proposition 7 *The government chooses its intervention intensity $\vartheta_{\hat{N}}$ to maximize*

$$\sup_{\vartheta_{\hat{N}}} \quad \frac{\sigma_V^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2} - \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}$$
$$-\gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right],$$

where $\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}$ and $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ are given in Appendix B.

The social welfare derived in Proposition 7 highlights the two particular objectives for government intervention: 1) to improve price efficiency, which is given by the first term and represents a desire to guide the real investment of entrepreneurs; and 2) to reduce price volatility, which is given by the third term and represents a desire to reduce the risk premia faced by all groups. The second term, which is always greater than or equal to 1, is the inverse of the informational advantage of informed investors and is consequently an additional motive to reduce price volatility.

The intervention objective derived in Proposition 7 can be roughly interpreted as a weighted average of the desires to improve price efficiency and reduce price volatility, with the weight on improving efficiency decreasing in σ_y^2, the variance of entrepreneurs' project-specific noise. Intuitively, as project-specific noise becomes more uncertain, the information extracted from the asset price becomes less useful to entrepreneurs. Building on this intervention objective, Figure 4 depicts the boundary between the government-centric equilibrium and the fundamental-centric equilibrium on a plane of σ_y^2 and σ_N^2 with other parameter values given in Table I. As σ_y^2 rises, the government assigns a lower weight to improving price efficiency and a higher weight to reducing price volatility. Consequently, the government intervenes more aggressively and the market shifts from the fundamental-centric equilibrium to the government-centric equilibrium at a lower threshold of σ_N^2. By combining our earlier analysis of the fundamental-centric and government-centric equilibria, Figure 4 suggests that, as the government assigns a greater weight to reducing price volatility, the resulting lower price volatility may come at the expense of lower price efficiency. In other words, the government faces a trade-off between financial stability and price efficiency in choosing its intervention policy.

6 Further Discussions

Our model highlights that, when adopting policies that lean against noise traders in financial markets, a government faces a tension between ensuring financial stability and improving price efficiency. We believe that this tension represents a key challenge faced by policy makers in China's financial system. As summarized in Section 2, the Chinese government has frequently engaged in leaning against financial-market cycles either through direct trading or through broad policy interventions. Our model shows that such interventions are helpful in terms of preventing market breakdown, which may occur when noise-trader risk is sufficiently large and market participants are short-termist. Depending on the weights the government puts on reducing price volatility and improving price efficiency, however, its intervention may lead to drastically different market dynamics. In particular, our model shows that, with an objective that weighs heavily on reducing asset price volatility, government intervention is more likely to move the market into a government-centric equilibrium, in which investors all focus on speculation of noise in government policy while ignoring asset fundamentals.

Consequences of government intervention in China. The Chinese government has announced multiple goals for its financial policies, but has not provided clear weights on these goals. Nevertheless, it is widely believed by the public that maintaining financial stability is of paramount importance, followed by stimulating economic growth. There are extensive discussions about the consequences of these extensive countercyclical interventions. Many recognize that the interventions have been successful in reducing market fluctuations and ensuring financial stability. More relevant to our analysis, however, is that some commentators have also pointed to their potential adverse effects on market efficiency. Allen et al.

(2020) and Huang, Miao, and Wang (2019) argue that, while the massive stock purchases by the national team during the 2015 stock market crash helped alleviate downside risk, this benefit may have come at the cost of preventing market discovery of stock prices and exacerbating the deviation of prices from their fundamental values. Indeed, Dang, Li, and Wang (2020) show that, in the cross-section of stocks, the trading of the national team is associated with reduced informativeness of stock prices. More generally, Zhu (2016) argues that the Chinese government's heavy interventions in its financial system, strongly motivated by its urge to ensure financial and social stability, have created implicit guarantees to investors, leading to adverse incentives of risk-seeking without concerns about risks and fundamentals. For example, Zhu argues that the dearth of public-firm delistings from the stock exchanges, partly related to regulators' reluctance to upset shareholders and stakeholders of potentially distressed firms, has emboldened stock investors to ignore firms' fundamentals and instead speculate on rumors and fads. In addition, the lack of public defaults by firms, driven mainly by the government's frequent intervention to bail out troubled borrowers, has emboldened households to invest in opaque shadow-banking credit products, contributing in recent years to China's leverage boom.

Government commitment. In practice, governments may not have the ability to commit to an intervention strategy, and a time-inconsistency problem arises that reinforces the government-centric equilibrium. In this situation, the government may want to initially convince investors that it will not intervene too aggressively, in the hope of inducing them to acquire information about the asset fundamental. After investors have collected fundamental information, however, the government—even with a single objective of improving information efficiency—has incentive to change its intentions ex post and to trade more aggressively against noise traders than it initially promised. Rationally anticipating this opportunistic behavior by the government, investors would always choose to collect information about the government's future trading noise instead. In this way, the time-inconsistency problem may lead to the government-centric equilibrium, even when the government prefers the fundamental-centric outcome. In a related paper, Brunnermeier, Sockin, and Xiong (2017) explore this time-inconsistency problem in the context of China's financial reform.

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Appendix A Deriving Equilibrium with Information Frictions and Government Intervention

In this Appendix, we derive the equilibrium with information frictions and government intervention in several steps. We assume that the economy is initialized from its stationary equilibrium, in which all conditional variances from learning have reached their deterministic steady state and the coefficients in prices and policies are time homogeneous.

We first consider the case without government intervention. We begin, as in the main text, by conjecturing a linear equilibrium price function:

$$P_t = p_{\hat{V}} \hat{V}_{t+1}^M + p_V \left(V_{t+1} - \hat{V}_{t+1}^M \right) + p_N N_t.$$

Importantly, we recognize that it must be the case that $p_{\hat{V}} = \frac{1}{R^f - \rho_V}$, since a unit shift in V_t must raise the discounted present value of future cash flows by $\frac{1}{R^f - \rho_V}$.

We first state several properties of the linear equilibrium without government intervention. We defer the derivation of the noisy rational expectations equilibrium to the case with government intervention, which is the more general case.

Proposition A1 *In the presence of informational frictions, the coefficient on the fundamental V, p_V, is less that $p_{\hat{V}}$, the coefficient on noise trading, p_N, is more positive. In addition, market breakdown occurs at a lower value of σ_N, σ_N^{**}, such that $\sigma_N^{**} \geq \sigma_N^*$, where σ_N^* is given in (1).*

In the presence of informational frictions, investors systematically underreact to information about the fundamental in prices (since $p_V < p_{\hat{V}}$) and overreact to noise. In addition, market breakdown occurs at lower levels of noise-trading variance than with perfect information. Since informational frictions introduce additional return volatility, investors require a higher risk premium to accommodate noise traders for the same level of noise-trader risk, σ_N^2. As a result, the critical value at which investors demand too high a risk premium to accommodate noise traders occurs at a smaller σ_N^2.

In the special case in which the fundamental, V, is i.i.d. ($\rho_V = 0$), we can express the condition for breakdown implicitly as

$$R^f < 2\gamma\sigma_N \sqrt{\sigma_D^2 + \left(\frac{1}{R^f}\right)^2 \left(\sigma_V^2 + \left((R^f)^2 - \frac{\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right) \frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV} + \tau_s^{-1}}\right)},$$

which reveals that uncertainty about V, parameterized through the posterior conditional variance of beliefs, $\Sigma^{M,VV}$, effectively raises the volatility of the fundamental from σ_V^2 to $\sigma_V^2 + \left(\left(R^f\right)^2 - \frac{\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}\right) \frac{\Sigma^{M,VV}\tau_s^{-1}}{\Sigma^{M,VV}+\tau_s^{-1}}$. There is both a direct effect that, for a fixed $\Sigma^{M,VV}$, the critical σ_N that leads to market breakdown falls, and an indirect effect that an increase in σ_N also increases $\Sigma^{M,VV}$. We can also establish that price volatility is higher, and price informativeness lower, with informational frictions when σ_N is sufficiently large.

Proposition A2 *In the special case that $\rho_V = 0$, price volatility is higher, and price informativeness is lower, in the presence of informational frictions when σ_N is sufficiently large.*

Having characterized the noisy rational expectations equilibrium without the government, we now consider the case with government intervention. We again conjecture a linear equilibrium price function:

$$P_t = p_{\hat{V}} \hat{V}_{t+1}^M + p_{\hat{G}} \hat{G}_{t+1}^M + p_V \left(V_{t+1} - \hat{V}_{t+1}^M \right) + p_G \left(G_{t+1} - \hat{G}_{t+1}^M \right) + p_g G_t + p_N N_t.$$

Importantly, we recognize that it must be the case that $p_{\hat{V}} = \frac{1}{R^f - \rho_V}$, since a unit shift in V_t must raise the discounted present value of future cash flows by $\frac{1}{R^f - \rho_V}$.

We now construct the equilibrium in several steps. We first solve for the learning processes of the government and investors, which begin with an intermediate step of deriving the beliefs from the perspective of the market that has access only to public information. Given the market's beliefs, which we can define recursively with the Kalman filter, we can construct the conditional posterior beliefs of the government and the posterior beliefs of each investor by applying Bayes' Rule to the market's beliefs given the private signal of each investor. We then solve for the optimal trading and information acquisition policies of the investors. Imposing market clearing, we can then express the government's objective in terms of the equilibrium objects we derive from learning.

Appendix A.1 Equilibrium Beliefs

In this subsection, we characterize the learning processes of the government and the investors. As we will see, it will be convenient to first derive the market's posterior beliefs about V_{t+1}, N_t, and G_{t+1}, respectively, which are Gaussian with conditional mean $\left(\hat{V}_{t+1}^M, \hat{N}_t^M, \hat{G}_{t+1}^M \right) = E\left[(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^M \right]$ and conditional variance $\Sigma_t^M = Var \left[\begin{bmatrix} V_{t+1} \\ N_t \\ G_{t+1} \end{bmatrix} \mid \mathcal{F}_t^M \right]$. Importantly, the market faces strategic uncertainty over the government's action as a result of the noise in the government's trading. As such, one must form expectations about this noise both for extracting information from prices and for understanding price dynamics and portfolio choice.

To solve for the market beliefs, we first construct the innovation process η_t^M for the asset price from the perspective of the market:

$$\begin{aligned} \eta_t^M &= P_t - \left(p_{\hat{V}} - p_V \right) \hat{V}_{t+1}^M - \left(p_{\hat{G}} - p_G \right) \hat{G}_{t+1}^M - p_g G_t \\ &= p_V V_{t+1} + p_G G_{t+1} + p_N N_t. \end{aligned}$$

Given that the investors and the government do not observe G_{t+1} (the next-period government noise), they must account for it in their learning.

Importantly, the asset price P_t and the innovation process η_t^M contain the same information, such that $\mathcal{F}_t^M = \sigma\left(\left\{ D_s, \eta_s^M, G_t \right\}_{s \le t} \right)$. Since the market's posterior about V_{t+1}

will be Gaussian, we need only specify the laws of motion for the conditional expectation $\left(\hat{V}_{t+1}^{M}, \hat{N}_{t}^{M}, \hat{G}_{t+1}^{M}\right)$ and the conditional variance Σ_t^M. As is standard with a Gaussian information structure, these estimates are governed by the Kalman filter. As a result of learning from prices, the beliefs of the market about V_{t+1}, N_t, and G_{t+1} will be correlated ex post after observing the asset price. We summarize this result in the following proposition.

Proposition A3 *Given the normal prior* $(V_0, N_0) \sim \mathcal{N}\left(\left(V, \bar{N}\right), \Sigma_0\right)$ *and* $G_0 \sim \mathcal{N}\left(0, \sigma_G^2\right)$, *the posterior market beliefs are Gaussian* $(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^M \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^{M}, \hat{N}_{t}^{M}, \hat{G}_{t+1}^{M}\right), \Sigma_{t+1}^M\right)$, *where the filtered estimates* $\left(\hat{V}_{t+1}^{M}, \hat{N}_{t}^{M}, \hat{G}_{t+1}^{M}\right)$ *follow the stochastic difference equations*

$$\begin{bmatrix} \hat{V}_{t+1}^{M} \\ \hat{N}_{t}^{M} \\ \hat{G}_{t+1}^{M} \\ G_t \end{bmatrix} = \begin{bmatrix} \rho_V \hat{V}_t^M \\ 0 \\ \hat{G}_{t|t-1}^{M} \\ G_{t-1} \end{bmatrix} + \mathbf{K}_t^M \begin{bmatrix} D_t - \hat{V}_t^M \\ \eta_t^M - p_V \rho_V \hat{V}_t^M \\ G_t - G_{t|t-1} \end{bmatrix},$$

and the conditional variance Σ_t^M *follows a deterministic induction equation. The market's posterior expectations of* V_{t+1}, N_t, *and* G_{t+1} *are related through*

$$p_V V_{t+1} + p_G G_{t+1} + p_N N_t = p_V \hat{V}_{t+1}^M + p_G \hat{G}_{t+1}^M + p_N \hat{N}_t^M.$$

Importantly, when the market tries to extract information from the price, market participants realize that the price innovations η_t^M contain the government trading noise G_{t+1}. As such, they must take into account the information content in the government noise when learning from the price and must form expectations about G_{t+1}. Through this channel, the path dependence of the government noise feeds into the market's beliefs, and the market has incentives to forecast the future noise in the government's trading.

Since investors learn through Bayesian updating, we can update their beliefs sequentially by beginning with the market beliefs, based on the coarser information set $\mathcal{F}_t^M$, and then updating the market beliefs with the private signals of investor i $\left(s_t^i, g_t^i\right)$. Given that the market posterior beliefs and investor private signals are Gaussian, this second updating process again takes the form of a linear updating rule. We summarize these steps in the following proposition.

Proposition A4 *Given the market beliefs, the conditional beliefs of investor* i *are also Gaussian* $(V_{t+1}, N_t, G_{t+1}) \mid \mathcal{F}_t^i \sim \mathcal{N}\left(\left(\hat{V}_{t+1}^{i}, \hat{N}_{t}^{i}, \hat{G}_{t+1|t}^{i}\right), \Sigma_t^s(i)\right)$, *where*

$$\begin{bmatrix} \hat{V}_{t+1}^{i} \\ \hat{N}_{t}^{i} \\ \hat{G}_{t+1}^{i} \end{bmatrix} = \begin{bmatrix} \hat{V}_{t+1}^{M} \\ \hat{N}_{t}^{M} \\ \hat{G}_{t+1}^{M} \end{bmatrix} + \Gamma_t' \begin{bmatrix} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{bmatrix},$$

and $\Sigma_t^s(i)$ *is related to* Σ_t^M *through a linear updating rule.*

Since the government does not observe any private information, its conditional posterior beliefs align with those of the market. In what follows, we focus on the covariance-stationary limit of the Kalman filter, after initial conditions have diminished and the conditional variances of beliefs have converged to their deterministic, steady state. The following corollary establishes that such a steady state exists.

Proposition A5 *There exists a covariance-stationary equilibrium, in which the conditional variance of the market beliefs has a deterministic steady state. Given this steady state, the beliefs of investors are also covariance-stationary.*

Having characterized learning by investors and the government in this economy, we now turn to the optimal policies of investors.

Appendix A.2 Investment and Information Acquisition Policies

We now examine the optimal policies of an individual investor i at time t who takes the intervention policy of the government as given. Given the CARA-normal structure of each investor's problem, the separation principle applies and we can separate the investor's learning process about (V_{t+1}, N_t, G_{t+1}) from his optimal trading policy. To derive the optimal investment policy, it is convenient to decompose the excess asset return as

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \phi' \varepsilon_{t+1}^M = \varsigma \Psi_t + \phi' \varepsilon_{t+1}^M,$$

where

$$\varepsilon_{t+1}^M = \left[\begin{array}{c} D_{t+1} - \hat{V}_{t+1}^M \\ \eta_{t+1}^M - p_V \rho_V \hat{V}_{t+1}^M - p_g \hat{G}_{t+1}^M \\ G_{t+1} - \hat{G}_{t+1}^M \end{array} \right],$$

and $\varepsilon_{t+1}^M \sim N\left(\mathbf{0}_{3\times 1}, \Omega^M\right)$ from Proposition A3. We can then decompose the excess return based on the information set of the investor:

$$R_{t+1} = E\left[R_{t+1} \mid \mathcal{F}_t^i\right] + \phi' \varepsilon_{t+1}^{S,i},$$

where we can update $E\left[R_{t+1} \mid \mathcal{F}_t^i\right]$ from $E\left[R_{t+1} \mid \mathcal{F}_t^M\right]$ by the Bayes' Rule according to

$$\begin{aligned} & E\left[R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i\right] \\ = \; & E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + CoV\left[R_{t+1}, \left[\begin{array}{c} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{array} \right]' \mid \mathcal{F}_t^M \right] \\ & \cdot Var\left[\left[\begin{array}{c} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{array} \right] \mid \mathcal{F}_t^M\right]^{-1} \left[\begin{array}{c} s_t^i - E\left[s_t^i \mid \mathcal{F}_t^M\right] \\ g_t^i - E\left[g_t^i \mid \mathcal{F}_t^M\right] \end{array} \right] \\ = \; & \varsigma \Psi_t + \frac{\phi' \omega \left[\begin{array}{cc} \Sigma^{M,G_1G_1} + \left[\left(1 - a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1} \\ -\Sigma^{M,VG_1} & \Sigma^{M,VV} + \left(a^i \tau_s\right)^{-1} \end{array} \right]}{\left(\Sigma^{M,VV} + (a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}\right) - \left(\Sigma^{M,VG_1}\right)^2} \left[\begin{array}{c} s_t^i - \hat{V}_{t+1}^M \\ g_t^i - \hat{G}_{t+1}^M \end{array} \right]. \end{aligned}$$

This expression shows that the investor's private information in either s_t^i or g_t^i can help him better predict the excess asset return relative to the market-based information. Since the investor is myopic, his optimal trading strategy is to acquire a mean-variance efficient portfolio based on his beliefs. This is summarized in the following proposition.

Proposition A6 *Given the state vector* $\Psi_t = \left[\hat{V}^M_{t+1}, \hat{N}^M_t, G_t, \hat{G}^M_{t+1}\right]$ *and investor i's signals* s^i_t *and* g^i_t*, investor i's optimal investment policy* X^i_t *takes the following form:*

$$X^i_t = \frac{1}{\gamma}\frac{\varsigma\Psi_t + \frac{\phi'\omega\begin{bmatrix}\Sigma^{M,G_1G_1}+\left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1}\\ -\Sigma^{M,VG_1} & \Sigma^{M,VV}+\left(a^i\tau_s\right)^{-1}\end{bmatrix}\begin{bmatrix}s^i_t-\hat{V}^M_{t+1}\\ g^i_t-\hat{G}^M_{t+1}\end{bmatrix}}{\left(\Sigma^{M,VV}+(a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1}+[(1-a)\tau_g]^{-1}\right)-\left(\Sigma^{M,VG_1}\right)^2}}{\phi'\Omega^M\phi - \frac{\phi'\omega\begin{bmatrix}\Sigma^{M,G_1G_1}+\left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1}\\ -\Sigma^{M,VG_1} & \Sigma^{M,VV}+\left(a^i\tau_s\right)^{-1}\end{bmatrix}\omega'\phi}{\left(\Sigma^{M,VV}+(a\tau_s)^{-1}\right)\left(\Sigma^{M,G_1G_1}+[(1-a)\tau_g]^{-1}\right)-\left(\Sigma^{M,VG_1}\right)^2}},$$

the coefficients ς, ϕ, *and* ω *given in the Online Appendix.*

This proposition shows that both signals s^i_t and g^i_t help the investor predict the asset return over the public information, because they can be used to form better predictions of V_{t+1} and G_{t+1}, which determine the asset return in the subsequent period. The investor needs to choose between acquiring either s^i_t or g^i_t based on the ex ante market information:

$$\begin{aligned}
E\left[U^i_t \mid \mathcal{F}^M_{t-1}\right] &= \sup_{a^i_t\in\{0,1\}} -E\left\{E\left[\exp\left(-\gamma R^f\bar{W} - \frac{1}{2}\frac{E\left[R_{t+1}\mid\mathcal{F}^i_t\right]^2}{Var\left[R_{t+1}\mid\mathcal{F}^i_t\right]}\right)\middle|\mathcal{F}^M_t\right]\middle|\mathcal{F}^M_{t-1}\right\}\\
&= \sup_{a^i_t\in\{0,1\}} -\sqrt{\frac{\phi'\left(\Omega^M - M\left(a^i_t\right)\right)\phi}{\phi'\Omega^M\phi}}E\left\{\exp\left(-\gamma R^f\bar{W} - \frac{\frac{1}{2}\left(\varsigma\Psi_t\right)^2}{\phi'\Omega^M\phi}\right)\middle|\mathcal{F}^M_{t-1}\right\}\\
&= \sup_{a^i_t\in\{0,1\}} -\sqrt{\frac{\phi'\left(\Omega^M - M\left(a^i_t\right)\right)\phi}{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}}\exp\left(-\gamma R^f\bar{W} - \frac{\frac{1}{2}\left(\varsigma\varrho\Psi_{t-1}\right)^2}{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}\right),
\end{aligned}$$

by the properties of the moment-generating function of noncentral chi-squared random variables, where

$$\begin{aligned}
M\left(a^i\right) &= \frac{\omega\begin{bmatrix}\Sigma^{M,G_1G_1}+\left[\left(1-a^i\right)\tau_g\right]^{-1} & -\Sigma^{M,VG_1}\\ -\Sigma^{M,VG_1} & \Sigma^{M,VV}+\left(a^i\tau_s\right)^{-1}\end{bmatrix}\omega'}{\left(\Sigma^{M,VV}+\left(a^i\tau_s\right)^{-1}\right)\left(\Sigma^{M,G_1G_1}+\left[\left(1-a^i\right)\tau_g\right]^{-1}\right)-\left(\Sigma^{M,VG_1}\right)^2}\\
&= \omega\begin{bmatrix}\frac{1}{\Sigma^{M,VV}+(a\tau_s)^{-1}} & 0\\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+[(1-a)\tau_g]^{-1}}\end{bmatrix}\omega'.
\end{aligned}$$

Since $p_{\hat{V}} = \frac{1}{R-\rho_V}$, $\varsigma = \begin{bmatrix} 0 & -R^f p_N & p_g - R^f p_{\hat{G}} & -R^f p_g\end{bmatrix}$:

$$\varsigma\varrho\Psi_{t-1} = -R^f p_g G_{t-1},$$

and therefore

$$E\left[U^i_t \mid \mathcal{F}^M_{t-1}\right] = \sup_{a^i_t\in\{0,1\}}\sqrt{\frac{\phi'\left(\Omega^M - M\left(a^i_t\right)\right)\phi}{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}}\exp\left(-\gamma R^f\bar{W} - \frac{\frac{1}{2}\left(R^f p_g G_{t-1}\right)^2}{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'}\right),$$

where $\phi'\Omega^M\phi + \varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma' = E\left[Var\left(R_{t+1}\right) \mid \mathcal{F}_{t-1}^M\right] + Var\left(E\left[R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-1}^M\right)$. By the Law of Total Variance, this implies

$$E\left[Var\left(R_{t+1}\right) \mid \mathcal{F}_{t-1}^M\right] + Var\left(E\left[R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-1}^M\right) = Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right).$$

Consequently, since $\phi'\left(\Omega^M - M\left(a_t^i\right)\right)\phi = Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)$ and a_t^i is a binary choice,

$$E\left[U_t^i \mid \mathcal{F}_{t-1}^M\right] = -\max_{a_t^i \in \{0,1\}} \sqrt{\frac{Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)}{Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right)}} \exp\left(-\gamma R^f \bar{W} - \frac{1}{2}\frac{\left(R^f p_g \hat{G}_t^M\right)^2}{Var\left(R_{t+1} \mid \mathcal{F}_{t-1}^M\right)}\right).$$

This is the expected utility of investor i based on the public information from the previous period. Importantly, we recognize that the investor's information acquisition choice is independent of the expectation with respect to $\mathcal{F}_{t-1}^M$. Intuitively, second moments are deterministic in a Gaussian framework, so the investor can perfectly anticipate the level of uncertainty he will face without knowing the specific realization of the common knowledge information vector Ψ_t tomorrow. We can further reduce this objective to

$$a^i = \arg\max_{a^i \in \{0,1\}} -\log\left\{\phi'\left[\Omega^M - M\left(a^i\right)\right]\phi\right\}, \tag{A1}$$

or, since log is a monotonic function and $\phi'\left[\Omega^M - M\left(a^i\right)\right]\phi = Var\left(R_{t+1} \mid \mathcal{F}_t^i\right)$,

$$a^i = \arg\sup_{a^i \in \{0,1\}} -Var\left(R_{t+1} \mid \mathcal{F}_t^M, a_t^i s_t^i + \left(1 - a_t^i\right) g_t^i, a_t^i\right).$$

Since the optimization objective involves only variances, which are covariance-stationary, the signal choice faced by the investors is time invariant. Intuitively, given the Gaussian price distribution and exponential utility for the investors, the benefit of more precise private information lies with the reduction in uncertainty over the excess asset return.

By substituting $M\left(a^i\right)$ into the optimization objective, we arrive at the following result.

Proposition A7 *Investor i chooses to acquire information about the asset fundamental V_{t+1} (i.e., $a^i = 1$) with probability λ:*

$$\lambda = \begin{cases} 1, & \text{if } Q < 0 \\ (0,1), & \text{if } Q = 0 \\ 0, & \text{if } Q > 0, \end{cases},$$

where

$$Q = \frac{CoV\left[R_{t+1}, G_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,G_1G_1} + \tau_g^{-1}} - \frac{CoV\left[R_{t+1}, V_{t+1} \mid \mathcal{F}_t^M\right]^2}{\Sigma^{M,VV} + \tau_s^{-1}}$$

is given explicitly in the Appendix, and $\lambda \in (0,1)$ is the mixing probability when the investor is indifferent between acquiring information about the asset fundamental or the government trading noise.

This proposition states that the investor chooses his signal to maximize his informational advantage over the market beliefs, based on the extent to which the signal reduces the conditional variance of the excess asset return. Importantly, this need not imply a preference for learning about V_{t+1} directly, since the government's future noise G_{t+1} also contributes to the overall variance of the excess asset return. The more the government's noise covaries with the unpredictable component of the asset return from the market's perspective, the more valuable this information is to the investors.[20] This is the partial equilibrium decision of each investor taking prices as given.

Appendix A.3 Market Clearing

Given the optimal policy for each investor from Proposition A7 and the government's trading policy in (3), imposing market clearing in the asset market leads to

$$\begin{aligned} 0 &= N_t + \lambda \frac{\varsigma\Psi_t + \frac{\phi'\omega}{\Sigma^{M,VV}+\tau_s^{-1}} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \left(V_{t+1} - \hat{V}^M_{t+1}\right)}{\gamma\phi' \left(\Omega^M - \omega \begin{bmatrix} \frac{1}{\Sigma^{M,VV}+\tau_s^{-1}} & 0 \\ 0 & 0 \end{bmatrix} \omega'\right)\phi} \\ &\quad + (1-\lambda) \frac{\varsigma\Psi_t + \frac{\phi'\omega}{\Sigma^{M,G_1G_1}+\tau_g^{-1}} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \left(G_{t+1} - \hat{G}^M_{t+1}\right)}{\gamma\phi' \left(\Omega^M - \omega \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+\tau_g^{-1}} \end{bmatrix} \omega'\right)\phi} - \vartheta_{\hat{N}}\hat{N}^M_t + \sqrt{\vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta}\,G_t, \end{aligned} \tag{A2}$$

where $\vartheta = \begin{bmatrix} 0 & \vartheta_{\hat{N}} & 0 & 0 \end{bmatrix}'$ and we have applied the Weak Law of Large Numbers (WLLN) that $\int_\chi s^i_t di = V_{t+1}$ and $\int_\chi g^i_t di = G_{t+1}$ over the arbitrary subset of the unit interval χ. In addition, we have recognized that $Var\left[\vartheta_{\hat{N}}\hat{N}^M \mid \mathcal{F}^M_{t-1}, \{a^i_t\}_i\right] = \vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta$. Following the insights of He and Wang (1995), we can express the market-clearing condition with a smaller, auxiliary state space given that expectations about V_{t+1} and N_t are linked through the stock price P_t. We now recognize that

$$\hat{N}^M_t = N_t + \frac{p_V}{p_N}\left(V_{t+1} - \hat{V}^M_{t+1}\right) + \frac{p_G}{p_N}\left(G_{t+1} - \hat{G}^M_{t+1}\right), \tag{A3}$$

from Proposition A3. This allows us to rewrite Ψ_t as the state vector $\tilde{\Psi}_t = [\hat{V}^M_{t+1}, \hat{G}^M_{t+1}, V_{t+1}, N_t, G_t, G_{t+1}]$.

Matching coefficients with our conjectured price function pins down the coefficients and confirms the linear equilibrium. Importantly, the coefficients are matched to the basis $\left\{\hat{V}^M_{t+1}, V_{t+1} - \hat{V}^M_{t+1}, \hat{G}^M_{t+1}, G_{t+1} - \hat{G}^M_{t+1}, G_t, N_t\right\}$ in accordance with our conjecture on the

[20] Since higher signal precision will reduce the conditional variance of the excess asset return but impact the expected return symmetrically because the signal is unbiased, the channel through which information acquisition affects portfolio returns is through reduction in uncertainty. Given that investors can take long or short positions without limit, the direction of the news surprise does not impact the information acquisition decision.

functional form of the asset price. This yields three conditions:

$$\begin{aligned}
0 &= -A\left(1+p_{\hat{V}}\left(\rho_V-R^f\right)\right),\\
\vartheta_{\hat{N}} &= 1-Ap_N R^f,\\
p_{\hat{G}} &= \frac{1}{R^f}p_g,
\end{aligned}$$

where

$$A=\frac{\lambda}{\gamma\phi'\left(\Omega^M-\omega\begin{bmatrix}\frac{1}{\Sigma^{M,VV}+\tau_s^{-1}} & 0\\ 0 & 0\end{bmatrix}\omega'\right)\phi}+\frac{1-\lambda}{\gamma\phi'\left(\Omega^M-\omega\begin{bmatrix}0 & 0\\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+\tau_g^{-1}}\end{bmatrix}\omega'\right)\phi}.$$

These conditions pin down the relationship between the government's trading policy and the price coefficients, and

$$-AR^f p_g+\sqrt{\vartheta'\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\vartheta} = 0, \tag{A4}$$

$$-\frac{p_V}{p_N}+\lambda\frac{\frac{\phi'\omega}{\Sigma^{M,VV}+\tau_s^{-1}}\begin{bmatrix}1\\0\end{bmatrix}}{\gamma\phi'\left(\Omega^M-\omega\begin{bmatrix}\frac{1}{\Sigma^{M,VV}+\tau_s^{-1}} & 0\\ 0 & 0\end{bmatrix}\omega'\right)\phi} = 0, \tag{A5}$$

$$-\frac{p_G}{p_N}+(1-\lambda)\frac{\frac{\phi'\omega}{\Sigma^{M,G_1G_1}+\tau_g^{-1}}\begin{bmatrix}0\\1\end{bmatrix}}{\gamma\phi'\left(\Omega^M-\omega\begin{bmatrix}0 & 0\\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+\tau_g^{-1}}\end{bmatrix}\omega'\right)\phi} = 0, \tag{A6}$$

which pin down p_g, p_V, and p_G and, consequently, the informativeness of the asset price given the loading on the noise-trading p_N. As one can see above, since the investors always take a neutral position on $\hat{V}_{t+1}^M$ (as it is common knowledge), the government also takes a neutral position by market clearing. The market-clearing condition (A4) reflects that the investors take an offsetting position to the noise G_t in the government's trading.

Since the investors determine the extent to which their private information about V_{t+1} and G_{t+1} is aggregated into the asset price, the government is limited in how it can impact price informativeness. This is reflected in the last two market-clearing conditions, (A5) and (A6). The second terms in these conditions are the intensities with which the investors trade on their private information about V_{t+1} and G_{t+1}, respectively. The first terms, $\frac{p_V}{p_N}$ and $\frac{p_G}{p_N}$, are the correlations of V_{t+1} and G_{t+1} with the perceived level of noise-trading $\hat{N}_t^M$, as can be seen from equation (A3). Since the government trades based on $\hat{N}_t^M$, it cannot completely separate its impact on the true level of noise-trading N_t in prices from its impact on V_{t+1} and G_{t+1}.

Given that the government internalizes its impact on prices when choosing its trading strategy $\vartheta_{\hat{N}}$, we can view its optimization problem as being over the choice of price coefficients $\{p_g, p_V, p_G, p_N\}$ in the price functional $P_t = p\left(\tilde{\Psi}_t\right)$, subject to the market-clearing conditions.

Appendix A.4 Computation of the Equilibrium

To compute equilibrium numerically, we follow the Kalman filter algorithm for the market beliefs outlined in Proposition A3 to find the stationary equilibrium. We then solve for the portfolio choice of each investor, impose the market-clearing conditions, and optimize the government's objective in choosing $\vartheta_{\hat{N}}$. Finally, we check each investor's information acquisition decision by computing the Q statistic to verify that the conjectured equilibrium is an equilibrium. We perform this optimization to search for both fundamental-centric ($\lambda = 1$) and government-centric ($\lambda = 0$) equilibria, as well as mixing equilibria ($\lambda \in (0,1)$), with the same equilibrium played at each date as consistent with covariance-stationarity.

Appendix B Welfare Analysis

In this Appendix, we further expand the model setting to analyze the welfare consequences of government intervention. The government is concerned with the welfare of four different types of agents in the economy: investors, noise traders, entrepreneurs, and taxpayers. For simplicity, we assume that these four groups are exclusive. All agents are risk averse and have CARA utility with common coefficient of absolute risk aversion γ. To minimize notation, we assume that asset markets are in a covariance-stationary equilibrium and, consequently, the government follows a stationary policy.

Investors. The first group, investors, follows directly from the main model in Section 4. At date t, they each take a position t in financial markets and, from our earlier analysis, garner expected utility:

$$U_t^i = -\exp\left(-\gamma R^f \bar{W} - \gamma E\left[X_t^i R_{t+1} \mid \mathcal{F}_t^i\right] + \frac{\gamma^2}{2}\left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]\right), \tag{A7}$$

where X_t^i can be decomposed as

$$X_t^i = X_t + \frac{1}{\gamma Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}\phi'\omega \begin{bmatrix} \frac{\left[a_t^i \tau_s\right]^{-1/2}}{\Sigma^{M,VV} + (a\tau_s)^{-1}} \varepsilon_t^{s,i} \\ \frac{\left[\left(1-a_t^i\right)\tau_g\right]^{-1/2}}{\Sigma^{M,G_1G_1} + \left[(1-a)\tau_g\right]^{-1}} \varepsilon_t^{g,i} \end{bmatrix},$$

and X_t is the aggregate position of informed investors and, by market clearing, equals $N_t - X_t^G$.

Noise traders. We next microfound noise traders as discretionary liquidity traders to incorporate their welfare from trading in the asset market. Similarly to Han, Tang, and Yang (2016), we assume that a continuum of liquidity traders needs to decide at date $t-1$ on whether to join trading in the asset market at date t, in order to receive a hedging benefit $B > 0$ in certainty equivalent utility. If liquidity trader j chooses to join the market, he needs to submit a market order at date t, which is given by

$$n_t^j = N_t + \sigma_n \varepsilon_{t+1}^j, \quad \varepsilon_{t+1}^j \sim iid\, \mathcal{N}(0,1),$$

where $\int_{\mathcal{D}} n_t^j dj = N_t$ by the WLLN on any measurable subset $\mathcal{D} \subseteq [0,1]$. If a trader chooses not to join the market, he earns a reservation utility, which we normalize to -1. At date $t-1$, liquidity trader j solves his expected utility from joining the market:

$$E\left[V_t^i \mid \mathcal{F}_{t-1}^M\right] = \max\left\{E\left[-\exp\left(-\gamma\left(B + n_t^i R_{t+1}\right)\right) \mid \mathcal{F}_{t-1}^M\right], -1\right\}.$$

We can express the excess return of the asset as

$$\begin{aligned} R_{t+1} &= D_{t+1} + \frac{1}{R^f - \rho_V} V_{t+2} - R^f \frac{1}{R^f - \rho_V} V_{t+1} + p_N N_{t+1} + R^f p_g \hat{G}_{t+2}^M + p_G \left(G_{t+2} - \hat{G}_{t+2}^M\right) \\ &\quad + \left(p_g - R^f p_G\right) G_{t+1} - R^f p_g G_t + \left(p_V\left(\rho_V - R^f\right) - 1\right)\left(V_{t+1} - \rho_V \hat{V}_t^M\right) \\ &\quad - R^f p_N N_t + \begin{bmatrix} 1 - p_V\left(R^f - \rho_V\right) \\ 0 \\ R^f p_G - p_g \\ 0 \end{bmatrix}' \mathbf{K}^M \varepsilon_{t+1}^M, \end{aligned}$$

where $\varepsilon_{t+1}^M = \begin{bmatrix} D_t - \hat{V}_t^M & \eta_t^M - p_V \rho_V \hat{V}_t^M & G_t - G_{t|t-1} \end{bmatrix}$. Since only terms in the last line are ex ante correlated with N_t, it follows that

$$E\left[V_t^i \mid \mathcal{F}_{t-1}^M\right] = -\exp(-\gamma B)\, E\left[\exp\left(\gamma A p_N N_t n_t^j\right) \mid \mathcal{F}_{t-1}^M\right],$$

where $p_N \geq 0$ and

$$A = R^f + \begin{bmatrix} p_V\left(R^f - \rho_V\right) - 1 \\ 0 \\ p_g - R^f p_G \\ 0 \end{bmatrix}' \mathbf{K}^M \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

By the property of the moment-generating function of the chi-square distribution,

$$E\left[V_t^i \mid \mathcal{F}_{t-1}^M\right] = \max\left\{-\exp\left(\frac{1}{2}\log\left(1 + 2\gamma A p_N \sigma_N^2\right) - \gamma B\right), -1\right\}.$$

Consequently, a liquidity trader at date $t-1$ will participate at date t if

$$B \geq \frac{1}{2\gamma}\log\left(1 + 2\gamma A p_N \sigma_N^2\right),$$

provided that $A p_N > -\frac{1}{2\gamma\sigma_N^2}$. Thus, for B sufficiently large, all liquidity traders will choose at $t-1$ to participate in the asset market at date t. Furthermore, since the asset price is covariance-stationary, the full measure of liquidity traders will participate at all dates.

For the government's welfare accounting, the expected utility of each liquidity trader at date t is

$$\begin{aligned} V_t^i &= E\left[-\exp\left(-\gamma\left(B + n_t^j R_{t+1}\right)\right) \mid \mathcal{F}_t^M\right] \qquad \text{(A8)} \\ &= -\exp\left(\gamma B - \gamma n_t^j E\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{\gamma^2}{2}\left(n_t^j\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]\right). \end{aligned}$$

Entrepreneurs. We now introduce a third group, entrepreneurs, who make investment decisions based on information extracted from the asset price. At date t, a continuum of ex ante identical, risk-averse entrepreneurs can invest in a risky project whose quality is positively correlated with $\varepsilon_t^V = V_t - \rho_V V_{t-1}$, the innovation in the fundamental of the traded asset. By investing in capital K_t at date t, the project provides a net profit at date $t+1$ of

$$Y_{t+1}^l = \beta \left(\varepsilon_{t+1}^V + \sigma_y \varepsilon_{t+1}^l \right) K_t,$$

where $\varepsilon_{t+1}^l \sim \mathcal{N}(0,1)$ is project-specific noise that is independent across entrepreneurs and σ_y^2 is the variance of the project-specific noise. As ε_{t+1}^V is not observable to entrepreneurs at t, they rely on the history of asset prices and dividends $\{D_s, P_s\}_{s \le t}$ contained in the public information set $\mathcal{F}_t^M$ to infer the value of V_{t+1} and ε_{t+1}^V.

An entrepreneur l chooses K_t at date t to maximize its expected utility Q_t^l :

$$\begin{aligned} Q_t^l &= \sup_{K_t} E\left[-\exp\left(-\gamma Y_{t+1}^l \right) \middle| \; \mathcal{F}_t^M \right] \\ &= \sup_{K_t} -\exp\left(-\gamma\beta E\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M \right] K_t + \frac{\gamma^2\beta^2}{2} Var\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M \right] K_t^2 \right). \end{aligned}$$

Given its posterior $V_{t+1} \mid \; \mathcal{F}_t^M \sim \mathcal{N}\left(\hat{V}_{t+1}^M, \Sigma^{M,VV} \right)$, its posterior for ε_{t+1}^V is

$$\varepsilon_{t+1}^V \mid \; \mathcal{F}_t^M \sim \mathcal{N}\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M, \left(1-\rho_V^2\right) \Sigma^{M,VV} \right).$$

It follows that all entrepreneurs choose the same optimal level of investment:

$$K_t = \frac{E\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M \right]}{\gamma\beta Var\left[\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \mid \mathcal{F}_t^M \right]} = \frac{1}{\gamma\beta} \frac{\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M}{\left(1-\rho_V^2\right) \Sigma^{M,VV} + \sigma_y^2}.$$

Then, the realized output Y_{t+1}^k is given by

$$Y_{t+1}^l = \frac{1}{\gamma} \frac{\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M}{\left(1-\rho_V^2\right) \Sigma^{M,VV} + \sigma_y^2} \left(\varepsilon_{t+1}^V + \varepsilon_{t+1}^l \right),$$

and the entrepreneur's expected utility is

$$Q_t^l = -\exp\left(-\frac{1}{2} \frac{\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M \right)^2}{\left(1-\rho_V^2\right) \Sigma^{M,VV} + \sigma_y^2} \right). \tag{A9}$$

Taxpayers. Finally, we include the fourth group, taxpayers, who are the residual claimants to the government and consequently receive its trading profit each period. At each date t, a new generation of taxpayers receives the profit from the government's trading at date t. Their expected utility as a group is

$$\begin{aligned} H_t &= E\left[-\exp\left(-\gamma X_t^G R_{t+1} \right) \mid \mathcal{F}_t^M \right] \\ &= -\exp\left(-\gamma X_t^G E\left[R_{t+1} \mid \mathcal{F}_t^M \right] + \frac{\gamma^2}{2} \left(X_t^G \right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M \right] \right). \end{aligned} \tag{A10}$$

Welfare function. We assume that the government adopts a variant of the Nash social welfare function, as in Kaneko and Nakamura (1979):

$$U_t^G\left(\vartheta_{\hat{N}}\right) = -\int_0^1 \log\left(-U_t^i\right) di - \int_0^1 \log\left(-V_t^j\right) dj - \int_0^1 \log\left(-Q_t^l\right) dl - \log\left(-H_t\right). \quad \text{(A11)}$$

This criterion is a monotonic transformation of the product of the utilities of all agents in the economy. It is an extension of the objective in the Nash bargaining solution for two players and the coalition Nash bargaining for N agents (Compte and Jehiel (2010)) to social choice theory. Similar to utilitarian welfare, this welfare criterion satisfies several desirable properties: Pareto optimality, independence of irrelevant alternatives, anonymity, and continuity (Kaneko and Nakamura (1979)), as well as independence of a common scale and a preference for equity (Moulin (2004)).

Substituting for U_t^i, V_t^j, and Q_t^k, we arrive at

$$\begin{aligned} U_t^G\left(\vartheta_{\hat{N}}\right) &= \gamma \int_0^1 X_t^i E\left[R_{t+1} \mid \mathcal{F}_t^i\right] di - \frac{\gamma^2}{2}\int_0^1 \left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] di - \gamma E\left[N_t R_{t+1} \mid \mathcal{F}_t^M\right] \\ & -\frac{\gamma^2}{2}\left(N_t^2+\sigma_n^2\right) Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \gamma E\left[X_t^G R_{t+1} \mid \mathcal{F}_t^M\right] \\ & -\frac{\gamma^2}{2}\left(X_t^G\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] + \frac{1}{2}\frac{\left(\hat{V}_{t+1}^M - \rho_V \hat{V}_t^M\right)^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2} + \gamma R^f \bar{W} - \gamma B\ , \end{aligned}$$

by noting that $\int_0^1 n_t^j dj = N_t$ and $\int_0^1 \left(n_t^j\right)^2 dj = N_t^2 + \sigma_n^2$ by the WLLN.

We assume that the government determines its intervention intensity $\vartheta_{\hat{N}}$ two periods ahead. That is, it chooses $\vartheta_{\hat{N}}$ for date t at date $t-2$. This timing reflects that the government cannot quickly adjust its intervention strategy in response to market conditions. The government has the public information set and chooses $\vartheta_{\hat{N}}$ to maximize its objective, by taking as given the information acquisition decision of informed investors. Since asset markets are covariance-stationary, the optimal information acquisition choice of informed investors at date $t-1$ who trade at date t is known to the government at date $t-2$.

By imposing the Law of Iterated Expectations, $\int_0^1 X_t^i di = X_t$, and market clearing, we recognize that

$$\begin{aligned} & E\left[\int_0^1 E\left[X_t^i R_{t+1} \mid \mathcal{F}_t^i\right] di - E\left[N_t R_{t+1} \mid \mathcal{F}_t^M\right] \mid \mathcal{F}_{t-2}^M\right] + E\left[X_t^G R_{t+1} \mid \mathcal{F}_{t-2}^M\right] \\ = \ & E\left[\left(\int_0^1 X_t^i di - N_t + X_t^G\right) R_{t+1} \mid \mathcal{F}_{t-2}^M\right] = 0, \end{aligned}$$

which simply indicates that trading is a zero-sum game between investors, noise traders, and the government. As a result, the social welfare is not affected by any group's expected trading gain, but rather by the second-moment terms:

$$\begin{aligned} E\left[U_t^G\left(\vartheta_{\hat{N}}\right) \mid \mathcal{F}_{t-2}^M\right] &= -\frac{\gamma^2}{2}\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] \\ &\quad -\frac{1}{2}\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}-\frac{\gamma^2}{2}\left(\sigma_N^2+\sigma_n^2\right)Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] \\ &\quad +\frac{1}{2}\frac{\sigma_V^2+\sigma_y^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2}+\gamma R^f\bar{W}-\gamma B. \end{aligned}$$

From our earlier derivation of X_t^i, we have

$$E\left[\gamma^2\left(X_t^i\right)^2 Var\left[R_{t+1} \mid \mathcal{F}_t^i\right] \mid \mathcal{F}_{t-2}^M\right]=E\left[\frac{E\left[R_{t+1} \mid \mathcal{F}_t^i\right]^2}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \mid \mathcal{F}_{t-2}^M\right]=\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]}-1,$$

and, in addition

$$E\left[\frac{\left(\hat{V}_{t+1}^M-\rho_V\hat{V}_t^M\right)^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2} \mid \mathcal{F}_{t-2}^M\right]=\frac{\sigma_V^2+\sigma_y^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2}-1.$$

Since $X_t^G=-\vartheta_{\hat{N}}\hat{N}_t^M+\sqrt{Var\left[\vartheta_{\hat{N}}\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]}G_t$ and G_t is observable at date t, we have

$$E\left[\left(X_t^G\right)^2 \mid \mathcal{F}_{t-2}^M\right]=\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)Var\left[\hat{N}_t^M \mid \mathcal{F}_{t-1}^M\right]=\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right).$$

As a result, the government's intervention objective is

$$\begin{aligned} \sup_{\vartheta_{\hat{N}}} \quad & \frac{\sigma_V^2}{\left(1-\rho_V^2\right)\Sigma^{M,VV}+\sigma_y^2}-\frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} \\ & -\gamma^2\left(\sigma_N^2+\sigma_n^2+\vartheta_{\hat{N}}^2\left(1+\sigma_G^2\right)\left(\sigma_N^2-\Sigma^{M,NN}\right)\right)Var\left[R_{t+1} \mid \mathcal{F}_t^M\right], \end{aligned}$$

where

$$\begin{aligned} \frac{Var\left[R_{t+1} \mid \mathcal{F}_{t-2}^M\right]}{Var\left[R_{t+1} \mid \mathcal{F}_t^i\right]} &= \frac{\phi'\Omega^M\phi+\varsigma\mathbf{K}^M\Omega^M\mathbf{K}^{M\prime}\varsigma'+\left(R^f p_g\right)^2\left(\sigma_G^2-\Sigma^{M,GG}\right)}{\phi'\left(\Omega^M-\omega\begin{bmatrix}-\frac{1}{\Sigma^{M,VV}+\left[a_i\tau_s\right]^{-1}} & 0 \\ 0 & \frac{1}{\Sigma^{M,G_1G_1}+\left[(1-a_i)\tau_g\right]^{-1}}\end{bmatrix}\omega'\right)\phi}, \\ Var\left[R_{t+1} \mid \mathcal{F}_t^M\right] &= \phi'\Omega^M\phi. \end{aligned}$$

There are three pieces in this objective. The first term is decreasing with the conditional variance of the market belief regarding the asset fundamental $\Sigma^{M,VV}$, which represents a desire for the government to increase price efficiency and guide the real investment of entrepreneurs. Also, note that this term is decreasing with σ_y^2, the variance of project-specific noise in entrepreneurs' projects. The second term is a variance ratio, which determines the informational advantage of informed investors. The third term is decreasing with the asset return variance, $Var\left[R_{t+1} \mid \mathcal{F}_t^M\right]$, which harms investors, liquidity traders, and taxpayers, who are all risk averse. This welfare criterion corresponds to the objective in the statement of Proposition 7.